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THE IDENTIFICATION (IN *SIMULINK*) OF AN ELASTIC SYSTEM

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ABSTRACT

In this paper is realized the identification of an elastic system with one degree of freedom, for which we experimentally pick up (in SIMULINK) the resonance curve. The system parameters (m, c, k) are determined using "the half-power points" from the curve.

1. Introduction

We consider an elastic system with one degree of freedom (fig. 1) which vibrates under the influence of

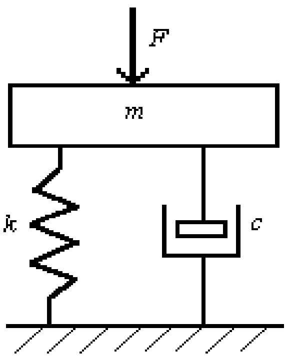

Fig. 1. The elastic system

the force $F(t) = F_O \sin \omega t$, according to the equation:

$$m\ddot{y} + c\dot{y} + k\,y = F \qquad (1)$$

The equation (1), written as:

$$\ddot{y} = \frac{F}{m} - \frac{c}{m}\dot{y} - \frac{k}{m}y\,, \qquad (2)$$

can be presented like in the *block diagram* from figure 2, where $f = F/m = f_O \sin \omega t$.

We consider the elastic system parameters of an electro-dynamic oscillator VED 1, realized and experimentally determined in Technical University from Timisoara:

$$m = 1.2\,;\ F_O = 15\,;\ p = \omega_r = 90\,;\ \zeta = 0.2\,, \qquad (3)$$

and calculate:

$$f_O = 15/1.2 = 12.5\,;\ p^2 = k/m \rightarrow k = 9720\,; \qquad (4)$$

$$\zeta = \frac{c}{2\sqrt{k\,m}} \rightarrow c = 43.2\,. \qquad (5)$$

For the equation (1) the solution is:

$$y = y_0 + y_p\,, \qquad (6)$$

where *the homogeneous* solution is:

$$y_0 = e^{-p\cdot\zeta\cdot t}(C_1 \cdot \cos p_1 t + C_2 \sin p_1 t), \qquad (7)$$

named **natural vibration** and:

$$p_1 = p\,; \qquad (8)$$

The *particular* solution is:

$$y_p = A \cdot \sin(\omega \cdot t - \theta)\,, \qquad (9)$$

named **forced vibration**.

Because of damping, the natural vibration quick vanishes (the transitory phase) and the forced vibration installs in the system from figure 1.

The constants A and θ in (9) are determined puting the condition that (9) verifies (1) [1]:

$$-m \cdot A \cdot \omega^2 (\cos\theta \cdot \sin\omega t - \sin\theta \cdot \cos\omega t) +$$
$$+ c \cdot A \cdot \omega (\cos\theta \cdot \cos\omega t + \sin\theta \cdot \sin\omega t) +$$
$$+ k \cdot A (\cos\theta \cdot \sin\omega t - \sin\theta \cdot \cos\omega t) \equiv F_0 \cdot \sin\omega t\,;$$

from identification it results the equations system::

$$-m \cdot A \cdot \omega^2 \cos\theta + c \cdot A \cdot \omega \sin\theta + k \cdot A \cos\theta = F_0$$
$$m \cdot A \cdot \omega^2 \sin\theta + c \cdot A \cdot \omega \cos\theta - k \cdot A \sin\theta = 0\,,$$

with the unknowns A and θ; it results:

$$A = \frac{F_o}{\sqrt{(k - m\omega^2)^2 + c^2\omega^2}} = \frac{3}{\sqrt{(9720 - 1.2\omega^2)^2 + 1866.24\,\omega^2}} \qquad (10)$$

In figure 3 is the graph representation, named the ***resonance curve***.

The maximum of A:

$$dA/d\omega = 0 \rightarrow \omega_r = p \cdot\,,$$

and

$$A_r = A(\omega_r) = \frac{F_o/k}{2\zeta\sqrt{1-\zeta^2}} \qquad (11)$$

Practically $\underline{\zeta << 1}$, therefore $\underline{\zeta^2 \cong 0}$, it results:

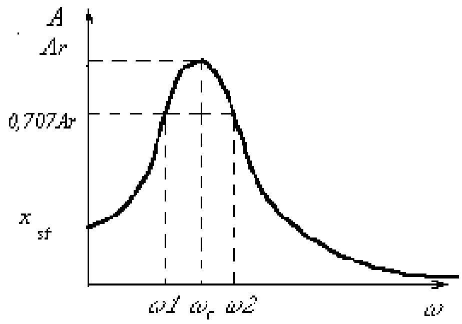

Fig. 3. The resonance curve

$$\omega_r \cong p\,, \qquad (12)$$

that is the *resonance pulsation* practically coincides with *natural pulsation* of the elastic system.

From (11): $A_r / (F_o / k) = 1/ (2\zeta)$,

whence

$$A_r = F_o / (2\, k\zeta) \;\rightarrow\; A_r = 3.86 \times 10^{-3}\ ; \qquad (13)$$

2. The SIMULINK schema [2]

The SIMULINK schema corresponding to the block diagram is presented in figure 4. The simulation results are those from table 1.

We will determine the elastic system parameters (*m, c, k*) in figure 1, having the possibility to experimentally determine (in SIMULINK) the *resonance curve* [1].

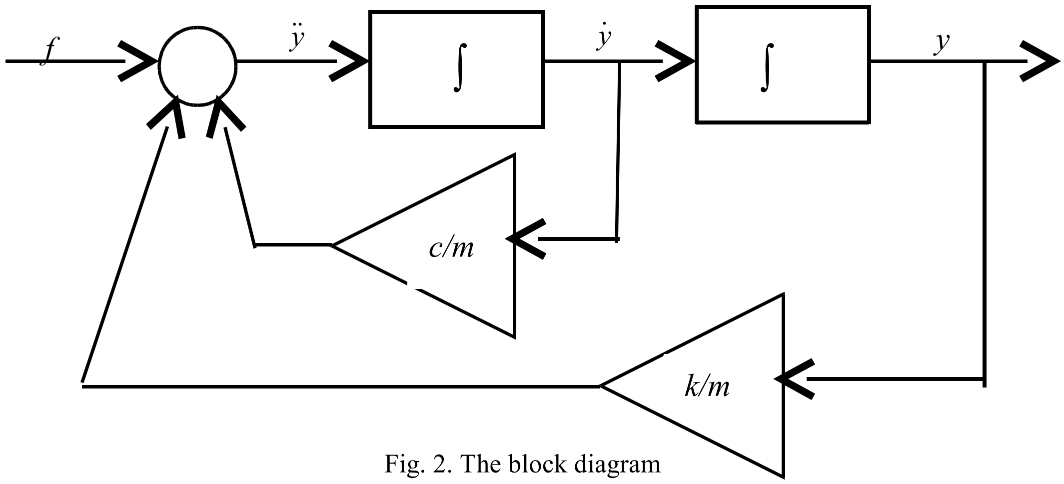

Fig. 2. The block diagram

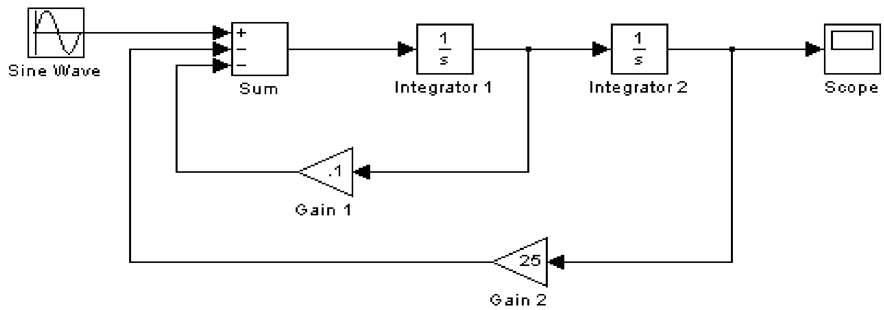

Fig. 4. The SIMULINK schema

Table 1

ω	0.05	1	10	30	50	60	**63**	70
$A \times 10^3$	1.5	1.5	1.6	1.7	2.1	2.5	**2.685**	3.02

ω	80	**90**	92	95	**105**	110	120
$A \times 10^3$	3.7	**3.8**	3.744	3.4	**2,68**	2.34	1.87

In table 1:

- it is verified that the maximum of $A(\omega)$ is reached for $\omega = \omega_r = p = 90$, which has the size $A_r = 3.8 \times 10^{-3}$, which differs a little from the value in (13);
- it is observed that *the half power points*, having the y-coordinates $0.707 \times A_r = 2.687 \times 10^{-3}$, correspond to the pulsations $\omega_1 = 63$ and $\omega_2 = 105$.

The degradation energy per cycle through viscous friction ($R = c\,\dot{y} = c\,A\,\omega \cdot \cos(\omega t - \theta)$) is equal with mechanical work of the R force:

$$E = \int_0^T R \cdot d\,y = \int_0^{2\pi/\omega} c\,A^2\,\omega^2 \cdot \cos^2(\omega t - \theta) \cdot d\,t = \pi\,c\,\omega\,A^2$$

For $A = A_r / \sqrt{2} = 0.707\,x\,A_r \rightarrow E' = E / 2$,

that is the dissipation of energy per cycle equal with half from those corresponding to resonance. This is the reason that the points having the y-coordinates $0.707\,x\,A_r$ named "the half power points" and we find them from condition $A_r / \sqrt{2} = A$, that is:

$$\frac{F_o / k}{2\sqrt{2}\,\zeta\,\sqrt{1-\zeta^2}} =$$

$$= \frac{F_0}{k} \cdot \frac{1}{\sqrt{(1-\omega^2/p^2)^2 + (2\zeta)^2\,(\omega/p)^2}} \quad (14)$$

We note:

$$\eta = \omega / p, \quad (15)$$

and from (14) it results the equation:

$$\eta^4 - 2\,(1 - 2\,\zeta^2)\,\eta^2 + 1 - 8\,\zeta^2\,(1 - \zeta^2) = 0;$$

$$\eta_{1,2}^2 = 1 - 2\zeta^2 \mp 2\zeta\,\sqrt{1-\zeta^2},$$

and for small damping ($\zeta^2 \cong 0$):

$$\eta_{1,2} = \omega_{1,2} / p \cong \sqrt{1 \mp 2\zeta};$$

so: $\omega_1^2 = p^2\,(1 - 2\,\zeta)$; $\omega_2^2 = p^2\,(1 + 2\,\zeta)$,

of which we can calculate: $\zeta = \dfrac{\omega_2^2 - \omega_1^2}{4\,p^2} = 0.218$,

very closed with (3).

3. The identification

We add to the system from figure 1 a supplementary mass $\Delta\,m = 0.49$; *the new* elastic system has the mass $m' = m + \Delta\,m = 1.69$ and corresponds a schema in SIMULINK to it, analog to this from figure 4, the only modifications being: $Gain\ 1' = c / m' = 25.56$ and $Gain\ 2' = k / m' = 5751.5$.

The natural pulsation of the new elastic system is

$$p_s = \omega_{rs} = \sqrt{\frac{k}{m'}} = 75.8, \quad (16)$$

value that we have verified by drawing the *resonance curve*, proceeding analogous to the table 1.

From the relations (4) and (16) we deduce:

$$k = m\,p^2 = (m + \Delta\,m)\,p_s^2,$$

whence: $$m = \frac{\Delta m}{\left(\dfrac{p}{p_s}\right)^2 - 1} = \frac{0.49}{\left(\dfrac{90}{75.8}\right)^2 - 1} = 1.195$$

and $k = \omega_r^2\,m = 90^2\,x\,1.195 = 9679.5$;

$c = 2\,\zeta\,\sqrt{k\,m} = 2 \cdot 0.218 \cdot \sqrt{9679.5 \cdot 1.195} = 46.9.$

4. Conclusions

The last three results being very closed to the values from (3, 4 and 5), we deduce that the identification has succeeded.

References

[1] **Balan, G.** (1988) *Mecanica vibratiilor*, Universitatea din Galati
[2] MATLAB 6.5