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THE DYNAMICS OF THE RIGID BODY WITH PASSIVE HYDRAULICS LINKAGES

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ABSTRACT

The paper refers to the case of the rigid body subject to linkages materialised through linear hydraulics engines. We will settle the mathematic model of the rigid and will determine its kinetic and dynamic characteristics. We accentuate the dynamic characteristics of the hydraulic linkage and the mutual influences between the rigid body and the linkage. We include the passive hydraulic linkage into the group of holonomic linkages.

1. General considerations

In the classical, Newtonian, mechanics there is the notion of linkage, expressing the movement restrictions imposed on the rigid through its connexions to the external environment. The linkages studied in the classic mechanics will be named in this paper classic linkages and will refer to: simple support, joint, linkages and wire linkages. The linkages are considered rigid, so consequently they reduce the movement possibility of the solid. The development of the applications imposed by the contemporary human civilization determined us to consider other types of linkages, such as: linkages through cinematic, elastic chains; linkages through viscous-elastic supports; linkages through fluid environments; linkages through polyphase environments; etc, linkages that are no longer rigid, and offer some movement possibilities to the rigid on the direction of the linkage, due to its deformation.

These linkages are taken into consideration too in mechanics, being classified by Boltzman as rheonome linkages and by Hertz as nonholonomic linkages of type I or II. In the case of these linkages, the time is explicitly, in the expression of the linkage itself, or implicitly, through cinematic characteristics (speed, acceleration) corresponding to the point of the rigid subject to the linkage. In the case of these types of linkages various specific mathematic models have been developed, by combining some simple models (known) in various geometric structures which to model as exact as possible the real linkage.

This paper discusses a category of linkages, used especially lately, for which we suggest the generic name of *hydraulic linkage*. This type of linkage of the rigid is made by using hydraulic components (hydraulic linear or revolving engines), through a viscous hydraulic environment, characterised by a certain elasticity, viscosity and some hysteretic properties.

The application of this type of linkages is generally known, any classic hydraulic action being included here, especially the actions specific to the active hydraulic suspensions of machines, the hydraulics actuations of the movement simulators, actuation of Stewart platforms, and generally, the equipments with high mobility degree, with hydraulic sequential actuation, in which some of the linkages are active (moving), and some other are passive (with closed hydraulic circuits).

The *active* hydraulic linkages are actually *nonholonomic* linkages that depend on the speed of movement of the point of the rigid in which the linkage is situated.

The *passive* hydraulic linkages are actually holonomic linkages that depend on the movement of point of rigid where the linkage is situated.

In this last case, the passive hydraulic linkages act like elastic linkages, made through the hydraulic environment "*blocked*" in actuation. This type of linkage is the object of the present paper.

2. Formulation of the case.

The model was already presented in [1].

Fig.1 presents the physical model of the rigid body with six freedom degrees, submitted to some active and passive hydraulic linkages.

The momentary position of a point of the rigid, named M, is determined in reference to the fixed reference system $O_1x_1y_1z_1$, through the moment coordinates of the mass centre C, that is x_C, y_C, z_C and through the angular movements of the mobile system $\varphi_x, \varphi_y, \varphi_z$.

The linkage points of the rigid with the exterior through the hydraulic linkage, named N_i, will have the momentary position determined identically.

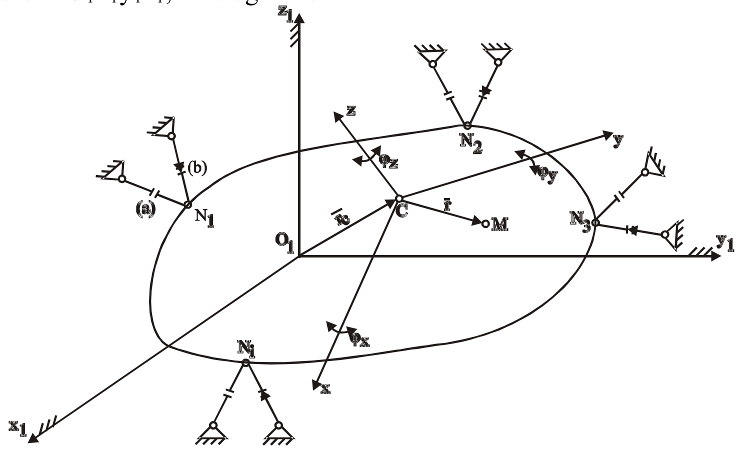


Fig.1 The model of the rigid with hydraulic linkages.
a) Passive hydraulic linkage; b) Active hydraulic linkage.

Expressing this dependency, using the matrix formalism, for the point M of the rigid, we get:

$$\{\Delta\} = \{r_c\} + [\varphi]\{r\} \quad (1)$$

where: $\{\Delta\} = \{\delta_x, \delta_y, \delta_z\}^T$;

$\{r_c\} = \{x_c, y_c, z_c\}^T; \{r\} = \{x, y, z\}^T$

$$[\varphi] = \begin{bmatrix} 0 & -\varphi_z & \varphi_y \\ \varphi_z & 0 & -\varphi_x \\ -\varphi_y & \varphi_z & 0 \end{bmatrix} \quad (2)$$

For a linkage point N_i:

$$\{\Delta_i\} = \{r_c\} + [\varphi]\{r_i\} \quad (3)$$

the speed of a linkage point N_i can be written, using the matrix formalism as:

$$\{v_i\} = \{v_c\} + [\omega]\{r_i\} \quad (4)$$

where: $\{v_i\} = \{\dot{x}_i; \dot{y}_i; \dot{z}_i\}^T$

$\{v_{ci}\} = \{\dot{x}_c; \dot{y}_c; \dot{z}_c\}^T$

$\{r_i\} = \{\dot{x}_i; \dot{y}_i; \dot{z}_i\}^T$

$$[\omega] = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_z & 0 \end{bmatrix} \quad (5)$$

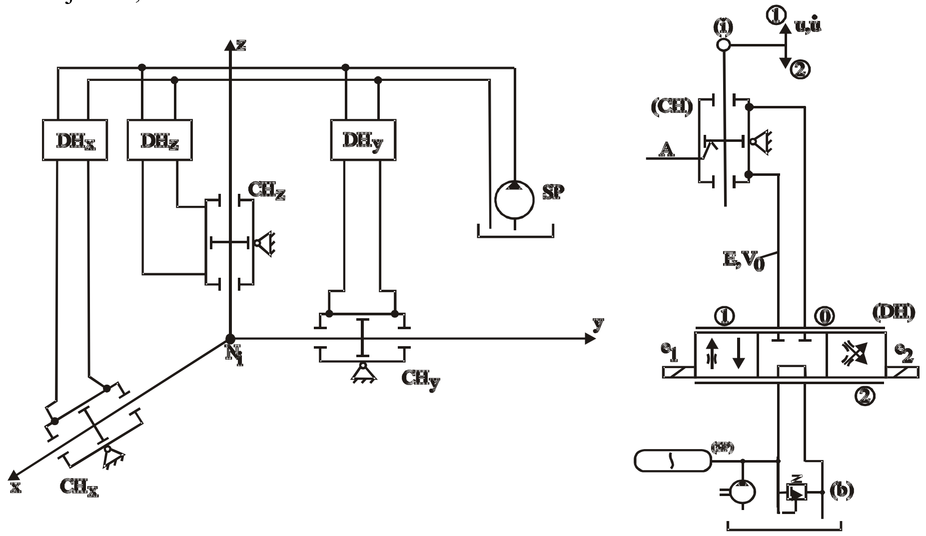
3. Defining the hydraulic linkage

We consider the point N_i of the rigid in which we apply a hydraulic linkage which acts on the *x*, *y*, *z* directions of the fixed reference, as in fig.2a.

The linkage may be either active on a direction or passive, according the position of the hydraulic switches (the DH proportional distributors), which can connect the hydraulic cylinders CH with the pressure source SP, or can discontinue this linkage. The DH switches command is made usually electrically, through an electronic command system, external to the mechanical system. In the case of flight simulators, which are part of the category of rigid-bodies analyzed in this work, the command of the switches is made by some process computers.

If we consider the unidirectional linkage in fig.2b, which acts on the direction (*u*), it will be a passive linkage if the switch (DH) is in the neutral position (O), (there are no e_1 and e_2 commands). If one of the e_1 and e_2 commands are made, the proportional distributor DH will be moved on position (1) or (2) the linkage being active, moving the point *i* of the rigid, on the direction *u*, with the speed $\dot{u}$, towards (1) or (2).

In this last case, the active hydraulic linkage is acting on the rigid body as an active force, applied directly, producing the rigid movement. The other linkages of the rigid, if they are passive, will act like viscous-elastic linkages, limiting the rigid's movement possibilities on the directions they are manifested. We must mention that the hydraulic linkages are connected to the rigid and to the external environment by spherical joints, considered ideal and without clearance. To approach the dynamical aspects we have to determine the energetic characteristics of the hydraulic linkage.



a) b)

Fig.2 The principle schematics of a hydraulic linkage
a) 3D linkage ; b) Unidirectional linkage;

We know that for a hydraulic circuit actuated with pressure p (fig.2b), the elementary energy of the hydraulic agent from the cylinder is given by the relation:

$$dV = p \cdot dv = Ap \cdot p \cdot du \qquad (6)$$

where:

dV [J] - elementary energy;

p [N/m^2] - the momentary pressure in the cylinder;

dv=Adu [m^3] - the elementary volume of the hydraulic agent in the cylinder;

A [m^2] - the surface of the cylinder's piston;

du [m] - the momentary liniar movement of the cylinder's rod.

According the equation of continuity of the hydraulic agent in the hydraulic system, we have:

$$Q = A\dot{u} + \alpha p + \beta \dot{p} \qquad (7)$$

where:

Q [m^3/sec] - the flow of hydraulic agent of the pressure source (SP) introduced in the hydraulic cylinder (CH);

$Q_m = A\dot{u}$ [m^3/sec] - the flow of hydraulic agent used in the cylinder to move the cylinder rod with linear speed $\dot{u}$;

$Q_p = \alpha \cdot p$ [m^3/sec] - the flow of hydraulic agent lost in the hydraulic system through leakages, losses proportional with momentary pressure in the hydraulic system;

$Q_h = \beta \cdot \dot{p}$ [m^3/sec] - the flow of hydraulic agent taken from the source through compensation of the compression of the hydraulic agent.

We mention that $\beta = V/E$ [m^4s^2/kg] represents the hydraulic capacity of the hydraulic agent from the system (of volume V_0) subject to compression effect, at the momentary pressure p.

The flow Q of hydraulic agent taken by cylinder CH will be (for the passive hydraulic linkage):

$$Q=0, \qquad (8)$$

From (8) and (7), considering the system loses non existent ($\alpha \cong 0$), we have:

$$A \cdot \dot{u} + \beta \cdot \dot{p} = 0 \qquad (9)$$

from where we have, through a simple calculation, the dependence of pressure on movement, respectively:

$$p = p_0 - \frac{A}{\beta} u \qquad (10)$$

where:

p_0 - represents the pressure from the cylinder at the statical equilibrium of the forces applied on the rigid;
u - the momentary movement of the point connected.
From (10) and (6) we have the expression of the potential energy of the passive support, as:

$$V = Ap_0u - \frac{A^2}{\beta}\frac{u^2}{2} = K_{iu}{}^*u - K_{iu}u^2 \quad (11)$$

4. Determining the characteristic quadratic form of the rigid

The kinetical energy of the rigid results from:

$$E = \frac{1}{2}\int_D v^2 dm \quad (12)$$

where *v*, results from the relation (4), which leads after development to the expresion, principially known:

$$\begin{aligned} 2E = {} & m(\dot{x}_c{}^2 + \dot{y}_c{}^2 + \dot{z}_c{}^2) + J_x\omega_x{}^2 + \\ & + J_y\omega_y{}^2 + J_z\omega_z{}^2 - 2J_{xy}\omega_x\omega_y - \\ & - 2J_{xz}\omega_x\omega_z - 2J_{yz}\omega_y\omega_z \end{aligned} \quad (13)$$

The potential energy is calculated under the hypothesis that its variation is integrally taken by the variation of the pressure potential energy of the passive hydraulic linkage, given by equation (11).
If we consider the movements of the rigid’s points connected by a passive hydraulic linkage, given by the relations (3), then the potential energy in the *i* order linkage, is:

$$\begin{aligned} V_i = {} & K_{ix}{}^*\delta_{ix} + K_{iy}{}^*\delta_{iy} + K_{iz}{}^*\delta_{iz} - \\ & - K_{ix}\delta_{ix}{}^2 - K_{iy}\delta_{iy}{}^2 - K_{iz}\delta_{iz}{}^2 \end{aligned} \quad (14)$$

For all the points of the rigid subject to passive hydraulic linkages, the total potential energy will be:

$$\begin{aligned} V = {} & \sum_i V_i = \sum_i K_{ix}{}^*\delta_{ix} + \sum_i K_{iy}{}^*\delta_{iy} + \\ & + \sum_i K_{iz}{}^*\delta_{iz} - \sum_i K_{ix}\delta_{ix}{}^2 - \\ & - \sum_i K_{iy}\delta_{iy}{}^2 - \sum_i K_{iz}\delta_{iz}{}^2 \end{aligned} \quad (15)$$

Considering the distortions given by (3) for each linkage point Ni and ordering the terms depending on the movement, we have:

$$\begin{aligned} V = {} & \sum_i K_{ix}{}^*x_c + \sum_i K_{iy}{}^*y_c + \\ & + \sum_i K_{iz}{}^*z_c - \sum_i K_{ix}x_c{}^2 - \\ & - \sum_i K_{iy}y_c{}^2 - \sum_i K_{iz}z_c{}^2 + \\ & + \sum_i (K_{iz}{}^*y_i - K_{iy}{}^*z_i)\varphi_x + \\ & + \sum_i (K_{ix}{}^*z_i - K_{iz}{}^*x_i)\varphi_y + \\ & + \sum_i (K_{iy}{}^*x_i - K_{ix}{}^*y_i)\varphi_z - \\ & - \sum_i (K_{iy}z_i{}^2 - K_{iz}y_i{}^2)\varphi_x{}^2 - \\ & - \sum_i (K_{iz}x_i{}^2 - K_{ix}z_i{}^2)\varphi_y{}^2 - \\ & - \sum_i (K_{ix}y_i{}^2 - K_{iy}x_i{}^2)\varphi_z{}^2 - \\ & - 2\sum_i K_{ix}z_ix_c\varphi_y - 2\sum_i K_{iy}z_iy_c\varphi_z - \\ & - 2\sum_i K_{iz}y_iz_c\varphi_x + 2\sum_i K_{ix}y_ix_c\varphi_z + \\ & + 2\sum_i K_{iy}z_iy_c\varphi_x + 2\sum_i K_{iz}x_iz_c\varphi_y + \\ & + 2\sum_i K_{iz}x_iy_i\varphi_x\varphi_y + \\ & + 2\sum_i K_{ix}y_iz_i\varphi_y\varphi_z + \\ & + 2\sum_i K_{iy}z_ix_i\varphi_z\varphi_x \end{aligned} \quad (15')$$

5. Determining the disturbing components

We consider that on the rigid act *j* disturbing concentrated forces and *k* disturbing couples.
The virtual mechanical work of the disturbing forces will have the expression:

$$\begin{aligned} \delta L_F = {} & \sum_J \bar{F}_J\delta\bar{r}_J = \sum_J F_{Jx}\delta x_J + \\ & + \sum_J F_{Jy}\delta y_J + \sum_J F_{Jz}\delta z_J \end{aligned} \quad (16)$$

Ordering the terms according to virtual movement ($x_c; y_c; z_c; \varphi_x; \varphi_y; \varphi_z$) we have the final expression of the virtual mechanical work of the disturbing forces:

$$\begin{aligned} \delta L_F = {} & \sum_J \bar{F}_J\delta x_c + \sum_J F_{Jx}\delta y_c + \sum_J F_{Jz}\delta z_c + \\ & + \sum_J (y_jF_{Jz} - z_jF_{Jy})\delta\varphi_x + \\ & + \sum_J (z_jF_{Jx} - x_jF_{Jz})\delta\varphi_y + \\ & + \sum_J (x_jF_{Jy} - y_jF_{Jx})\delta\varphi_z \end{aligned} \quad (17)$$

The virtual mechanical work of the torques directly applied to the rigid, has the expression:

$$\delta L_M = \sum_k \overline{M}_k \delta\varphi = = M_x \delta\varphi_J + M_{xy}\delta\varphi_y + M_z \delta\varphi_z \qquad (18)$$

The cumulated effect of the disturbing factors results by summing the relations (17) and (18), from where, after ordering the terms after the virtual movements results:

$$\begin{aligned}
&\delta L_x = (\sum_J F_{Jx})\delta x_c;\\
&\delta L_y = (\sum_J F_{Jy})\delta y_c;\\
&\delta L_z = (\sum_J F_{Jz})\delta z_c;\\
&\delta L_{\varphi x} = \left[\sum_J (y_j F_{Jz} - z_j F_{Jy}) + \sum_k M_{kx}\right]\delta\varphi_x; \quad (19)\\
&\delta L_{\varphi y} = \left[\sum_J (z_j F_{Jx} - x_j F_{Jz}) + \sum_k M_{ky}\right]\delta\varphi_y;\\
&\delta L_{\varphi z} = \left[\sum_J (x_j F_{Jy} - y_j F_{Jx}) + \sum_k M_{kz}\right]\delta\varphi_z
\end{aligned}$$

6. The movement equations of the rigid

For deducing the moving equations we use the Lagrange equations of type II, with the expression:

$$\frac{d}{dt}\left(\frac{\partial E}{\partial\{\dot{x}\}}\right) - \left(\frac{\partial E}{\partial\{x\}}\right) = \{Q\} \qquad (20)$$

where:

E - is the kinetic energy of the rigid, defined by relation (13)

$\{\dot{x}\} = \{\dot{x}_c, \dot{y}_c, \dot{z}_c, \dot{\varphi}_x, \dot{\varphi}_y, \dot{\varphi}_z\}^T$ - is the matrix associated to the generalised speeds of the rigid;

$\{x\} = \{x_c, y_c, z_c, \varphi_x, \varphi_y, \varphi_z\}^T$ - is the matrix associated to the generalised coordinates or the rigid;

$\{Q\} = \{\{Q^P\} + \{Q^F\}\}$ - are the generalised forces, where:

$\{Q^P\} = -\dfrac{\partial V}{\partial\{x\}}$ - represents the generalised forces of potential nature, in our case from the passive hydraulic linkages and

$\{Q^F\} = -\dfrac{\delta L_{(x)}}{\delta\{x\}}$ - represents the generalised disturbing forces.

From the equations (20), using the matrix formalism, we have the matrix equation:

$$[M_0]\{\dot{v}\} + [K_0]\{x\} = \{Q^F\} + \{R_0\} \qquad (21)$$

where:

$\{\dot{v}\} = \{\ddot{x}_c, \ddot{y}_c, \ddot{z}_c, \ddot{\varphi}_x, \ddot{\varphi}_y, \ddot{\varphi}_z\}^T$;

$\{x\} = \{x_c, y_c, z_c, \varphi_x, \varphi_y, \varphi_z\}^T$

$[M_0]$ - the inertia matrix of the rigid, which has the expression:

$$[M_0] = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0\\ 0 & m & 0 & 0 & 0 & 0\\ 0 & 0 & m & 0 & 0 & 0\\ 0 & 0 & 0 & J_x & -J_{xy} & -J_{xz}\\ 0 & 0 & 0 & -J_{yx} & J_y & -J_{yz}\\ 0 & 0 & 0 & -J_{zx} & -J_{zy} & J_z \end{bmatrix} \qquad (22)$$

$[K_0]$ - represents the rigidity matrix of the passive hydraulic linkages and has the following expression:

$$\begin{bmatrix}
\sum\frac{1}{V_{ox}} & 0 & 0 & 0 & \sum\frac{z_i}{V_{ox}} & -\sum\frac{y_i}{V_{ox}}\\
0 & \sum\frac{1}{V_{oy}} & 0 & -\sum\frac{z_i}{V_{oy}} & 0 & \sum\frac{x_i}{V_{oy}}\\
0 & 0 & \sum\frac{1}{V_{oz}} & \sum\frac{y_i}{V_{oz}} & -\sum\frac{x_i}{V_{oz}} & 0\\
0 & -\sum\frac{z_i}{V_{oy}} & \sum\frac{y_i}{V_{oz}} & \sum\left(\frac{z_i^2}{V_{oy}} - \frac{y_i^2}{V_{oz}}\right) & -\sum\frac{x_i y_i}{V_{oz}} & -\sum\frac{x_i z_i}{V_{oy}}\\
\sum\frac{z_i}{V_{ox}} & 0 & -\sum\frac{x_i}{V_{oz}} & -\sum\frac{x_i y_i}{V_{oz}} & \sum\left(\frac{x_i^2}{V_{oz}} - \frac{z_i^2}{V_{ox}}\right) & -\sum\frac{y_i z_i}{V_{ox}}\\
\sum\frac{y_i}{V_{ox}} & \sum\frac{x_i}{V_{oy}} & 0 & -\sum\frac{x_i z_i}{V_{oy}} & -\sum\frac{y_i z_i}{V_{ox}} & \sum\left(\frac{y_i^2}{V_{ox}} - \frac{x_i^2}{V_{oy}}\right)
\end{bmatrix} \qquad (23)$$

where:

E - represents the elasticity module of the hydraulic agent and $V_{ox}; V_{oy}; V_{oz}$ - represent the volumes of hydraulic agent subject to compressibility;

$\{Q^F\}$ - represents the matrix attached to the vector of generalised disturbing forces;

$$\{Q^F\} = \begin{Bmatrix}
\sum_J F_{Jx}\\
\sum_J F_{Jy}\\
\sum_J F_{Jz}\\
\sum_J (y_j F_{Jz} - z_j F_{Jy}) + \sum_k M_{kx}\\
\sum_J (z_j F_{Jx} - x_j F_{Jz}) + \sum_k M_{ky}\\
\sum_J (x_j F_{Jy} - y_j F_{Jx}) + \sum_k M_{kz}
\end{Bmatrix} \qquad (24)$$

$\{R_0\}$ - represents the matrix attached to the vector of hydrostatic forces corresponding to the static equilibrium of the rigid;

$$\{R_0\} = A\begin{Bmatrix} \sum p_{ox} \\ \sum p_{oy} \\ \sum p_{oz} \\ \sum (p_{oz}y_i - p_{oy}z_i) \\ \sum (p_{ox}z_i - p_{oz}x_i) \\ \sum (p_{oy}x_i - p_{ox}y_i) \end{Bmatrix} \quad (25)$$

The matrix (22); (23); (24) and (25) can also be written, underlining the significance of the component terms:

$$[M_0] = \begin{bmatrix} [M] & [0] \\ [0] & [J] \end{bmatrix} \quad (26)$$

where:

$[M]$ - represents the matrix associated to the mass of the rigid;

$[J]$ - the matrix of the inertial tensor of the rigid;

$[0]_{3x3}$ -where:

$[M]$ - represents the matrix associated to the mass of the rigid;

$[J]$ - the matrix of the inertial tensor of the rigid;

$[0]_{3x3}$ - the null matrix;

$$[K_0] = = A^2\begin{bmatrix} \sum_i [C_{Thi}] & -\sum_i [C_{Thi}][r_i] \\ \sum_i [C_{Thi}][r_i] & -\sum_i [r_i][C_{Thi}][r_i] \end{bmatrix} = = -A^2\begin{bmatrix} [C_{Th}] & -[H] \\ [H] & -[L] \end{bmatrix} \quad (27)$$

$[C_{Th}]$ - represents the matrix associated to the total hydraulic capacity of the hydraulic linkages circuit;

$[r_i]$ - the matrix associated to the vector of the point of application of the hydraulic linkages;

$$[H] = \sum_i [C_{Thi}][r_i];$$

$$[L] = \sum_i [r_i][C_{Thi}][r_i];$$

$$\{Q^F\} = \{\{f\};\{m\}\}^T;$$

$$\{R_0\} = A\{\{\tau\};\{T\}\};$$

the differential equation (21) becomes:

$$\begin{bmatrix} [M] & [0] \\ [0] & [J] \end{bmatrix}\begin{Bmatrix} \{\ddot{x}\} \\ \{\ddot{\varphi}\} \end{Bmatrix} - - A^2\begin{bmatrix} [C_{Th}] & -[H] \\ [H] & -[L] \end{bmatrix}\begin{Bmatrix} \{x\} \\ \{\varphi\} \end{Bmatrix} = = \begin{Bmatrix} \{f\} \\ \{m\} \end{Bmatrix} + A\begin{Bmatrix} \{\tau\} \\ \{T\} \end{Bmatrix} \quad (28)$$

equation that better represent the linear and angular movement of the analysed rigid under the action of disturbing factors.

7. Conclusions

The dynamic analyses undertaken in this paper on the rigid body with passive hydrodynamic linkages reflects that the rigid in this situation acts as a rigid subject to elastic linkages, which was expected.

The characteristics of the hydraulic linkage that determines the dynamic behaviour of the rigid body are the hydraulic capacity of the hydraulic circuits.

When there are no disturbing factors $\{f\} = \{0\}$, $\{m\} = \{0\}$ and the hydraulic system of the passive linkages is dynamically equilibrated, $\{\tau\} = \{0\}$, $\{T\} = \{0\}$, from (21) and (28) equations we get the free vibrations of the rigid body subject to passive hydraulic linkages, whose own pulsations can be determined from the characteristic equation:

$$\det\left|-p^2[M_0] + [K_0]\right| = 0 \quad (29)$$

The aspects underlined in this paper may be used to analyse the possibility to dissipate the earthquake energy transmitted to buildings from the seismic waves and also the possibility to dissipate the energy transmitted to the environment by technological equipment that generates vibration and shocks.

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