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MATHEMATICAL MODELS FOR THE WHOLE TRACTION MECHANICAL SYSTEM OF THE TECHNOLOGICAL SELF-PROPELLED MACHINES

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ABSTRACT

The paper achieves mathematical models of the integral traction mechanical system, that is the system achieved only with gearings organized in the transmission components , gearboxes, card differentials transmissions, rolling systems on wheels or caterpillars. It was achieved the mathematical models for the self-propelled equipments on tires and caterpillars. In the paper differential equations describing the dynamic traction phenomena are written and are numerical modeled these systems. It has been seen that appear phenomena of resonance produced by excitation through the run away, resonances inducted in all the structural components: thermic engine, traction system equipment.

1. Introduction

The answer at the kinematic excitation of the traction system, produced by the run away, depends on tensional by the excited system, therefore by the structural composition of draft system analysis. The system of excitation steps in the model through the dynamic features which inducts the dynamic model (movement of excitation, speed of excitation, acceleration of excitation, etc). At excitation produced by the run away, the traction system of the equipment, answers depending on tensional [3]. The mathematical models for the suggested traction systems for analysis will be elaborated.

2. The dynamic models and the mathematical of STIM

2. 1. Model For the self-propelled equipments on tire

It is considered an equipment of massage M of which system of movement is achieved on tire, the driving wheels being the wheels back. The transmission of movement is achieved by the mechanic, utilizing a gearbox, card a drive, differential, driving wheels. The front wheels of the equipment have just the role of direction and not traction. (Fig. 1).

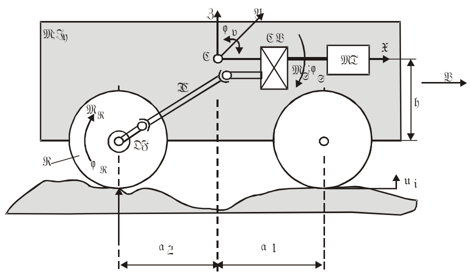

Fig. 1. The physical model of the equipment on tire.
MT-thermic engine; CV- gearbox; TC-cardan drive; DF-differential; R-the wheel with tire.

The model of draft system by-path realizations hypotheses as the: I am to eliminate the clutches, to load dynamics realized simultaneously by-groups of tire the face, without taking count the individual shipment tire; I considered characteristics viscous-elastic of tire on the horizontal, vertical and torque direction; the moment of adherence of the organ of thing the bed bearer (M_A) he is me constant, the bed bearer to considers rigid; the system the draft mechanic acts just the group of wheels back, the front wheels are of direction [1], [2].

The aim of the model is emphasized dynamic influential dislevels run away about transmission of movement mechanical the technological equipment on its tires or caterpillars, what moves with speed (v) across the dislevels of the road (u_i).

2. 1. 2. The modeling of the movement of the equipment in conditions of adherence

Just as it arises from fig. 2, the movement of the equipment with variable speed v is possible in the conditions of the adherence with the run away, below the act of draft force (F_0) form of the driving wheels, of the structure of the equipment.

It is remarked the fact that the induct of the motor moment at structure of the equipment as the draft force is achieved by the structure viscous-elastic of the tires demonstrated through the rigidity and the factor of amortization estimate on the movement direction of the equipment. Fall-backing on structure of the equipment from nr. 1, which we represent with active and resistant force, obtained the model dynamic from fig. 2.

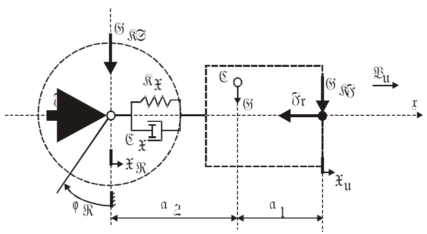

Fig. 2. The dynamic model of axial movement of the equipment

The differential equation describes the dynamic process of rectilinear movement of the equipment, is:

$$M\ddot{X}_R + c_x\left(\dot{X}_R - \dot{X}_U\right) + k_x\left(X_R - X_U\right) = F_0 - F_r \quad (1)$$

where: M - massage total the equipment; c_x - the factor of amortization of tire on the axis x; k_x -the tire rigidity on the direction x; X_R - the axial movement center driving wheels; X_U - the axial movement of the equipment; F_0 - traction force; F_r - resistant force inducted in the structure of the equipment to the run of the face total adhesive, put in the obvious by (3) between movements, speeds and angles exist the relations:

$$X_R = r_D\varphi_R, X_U = r_D\varphi_U, v_U = r_D\dot{\varphi}_U \quad (2)$$

which replayed in the relation (2) multiplication with r_D, give the equation:

$$J_U\ddot{\varphi}_R + c_\varphi\left(\dot{\varphi}_R - \dot{\varphi}_U\right) + k_\varphi\left(\varphi_R - \varphi_U\right) = M_k \quad ; \quad (3)$$

where: $J_U = M \cdot r_D^2$ - it represents the moment of inertia of the equipment boiled down to the axle wheels engine; $k_\varphi = k_x r_D^2$ - it represents the angular rigidity tire; $c_\varphi = c_x r_D^2$ - it represents the coefficient of angular amortization of tire; M_k -the active moment boiled down to the axle wheels, what to produce the acceleration of the equipment.

From the equation of kinetic energy wrote for the movement of the equipment and the equivalent turning $\left(\varphi_U\right)$ results the equality[5]:

$$\frac{Mv_U^2}{2} = \frac{J_U\dot{\varphi}_U^2}{2} \quad (4)$$

which in the condition $v_U = r_D\dot{\varphi}_U$, confirms the relation of the moment of inertia of the equipment reduced to the axle driven wheels.

2. 1. 3. The mathematical ultimate model

For whole equipment comprising: source of energy (thermic engine) transmission mechanical the driving wheel (tire) bed bearer structure equipment, the dynamic model can be considered a system a mechanic with three degrees of freedom, presented in the figure 3.

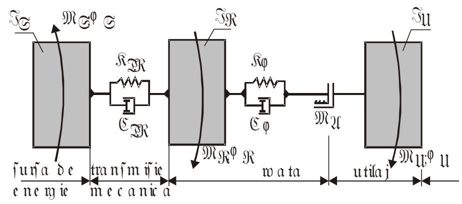

Fig. 3. The dynamic model of the equipment on tire

The equations describe the dynamic process of power transfer from the thermic engine to structure of the equipment by dint of the transmission of the mechanic and wheels with tire, results applying the principle D'alembert of the model from figure 3[1], [4].

It results:

$$\begin{cases} J_S\ddot{\varphi}_S + r^2 c_{TR}\left(\dot{\varphi}_S - i_{TR}\dot{\varphi}_R\right) + \\ + r^2 k_{TR}\left(\varphi_S - i_{TR}\varphi_R\right) = M_S; \\ J_R\ddot{\varphi}_R + c_\varphi\left(\dot{\varphi}_R - \dot{\varphi}_U\right) + k_\varphi\left(\varphi_R - \varphi_U\right) - \\ - i_{TR} r^2 c_{TR}\left(\dot{\varphi}_S - i_{TR}\dot{\varphi}_R\right) - \\ - i_{TR} r^2 k_{TR}\left(\varphi_S - i_{TR}\varphi_R\right) = -M_R; \\ J_T\ddot{\varphi}_U - c_\varphi\left(\dot{\varphi}_R - \dot{\varphi}_U\right) - \\ - k_\varphi\left(\varphi_R - \varphi_U\right) = -M_U; \end{cases} \quad (5)$$

which: J_S - the moment of inertia of thermic engine reduced to this axle; M_S -the moment of the source from external engine characteristic; J_R - the moment of inertia of the transmission, inclusively the driving wheels, reduced to the axle driving wheels; J_U - the moment of inertia of the equipment reduced to the axle driving wheels; J_T - of inertia of the reduced inclusive equipment the transmission and the driving wheels JT= JR+ JU; J_T -The momentary angle of rotation to the motor thermic axle; φ_R-The momentan angle of rotation to the axle driving wheels; $\varphi_U = X_u / r_D$ - the equivalent angle of movement of the equipment; c_{TR}, k_{TR}, c_φ, k_φ - the elastic features the and of amortization of the transmission and wheels; M_R - resistant moment the driving wheels; M_U - the resistant moment the advance front wheels (to push in his case 4x2 motive in the case 4x4). Interaction between the equipment in the movement, below the act motive couple, and the way on which this is moving is put the obvious by the resistant moments at the wheel, due to the nature of the rolling path M_{R1}, M_{U1}, quotient and the dynamic effects producted by the dislevels of path M_{R2}, M_{U2}. The reaction of resistance moment of the wheel is:

$$M_R = M_{R1} + M_{R2}; \tag{6}$$

For the driving wheels;

$$M_U = M_{U1} + M_{U2} \tag{7}$$

For the push wheels, respectively the resistances to the advance of the equipment:

$$M_{R1} = r_D . f . G_{KS}; M_{U1} = r_D \cdot f \cdot G_{KF}; \tag{8}$$

$$\begin{aligned} M_{R2} &= 2k_{z2}(z - u_2 - a_2\varphi_y).f.r_D; \\ M_{U2} &= 2k_{z1}\left(z - u_1 - a_1\varphi_y\right).f.r_D; \end{aligned} \tag{9}$$

Terms from the relations (6), (9) have significations described previously. About the relations of the moments (8), (9), is come back when is caused these expression resulted from the movement across the dislevelments rolling path.

If in the system (5) make-do the moment M_S with the proper contact obtained from feature of regulator, to order proper terms, results the mathematical model of dynamic behavior of the system STIM-P.

$$\begin{cases} J_S\ddot{\varphi}_S + c_{11}\dot{\varphi}_S + c_{12}\dot{\varphi}_R + k_{11}\varphi_S + \\ + k_{12}\varphi_R = \alpha_S; \\ J_R\ddot{\varphi}_R + c_{21}\dot{\varphi}_S + c_{22}\dot{\varphi}_R + c_{23}\dot{\varphi}_U + k_{31}\varphi_S + \\ + k_{32}\varphi_R + k_{33}\varphi_U = -M_R; \\ J_U\ddot{\varphi}_U + c_{32}\dot{\varphi}_R + c_{33}\dot{\varphi}_U + \\ + k_{32}\varphi_R + k_{33}\varphi_U = -M_U; \end{cases} \tag{10}$$

The unknown the model are: $\varphi_S, \varphi_R, \varphi_U$, and the parametric sizes of the model are: M_R and M_U, what they represents the perturbation factors inducted the in the system of kinematic excitation rolling path

2. 2. Model For the self-propelled equipments on caterpillars

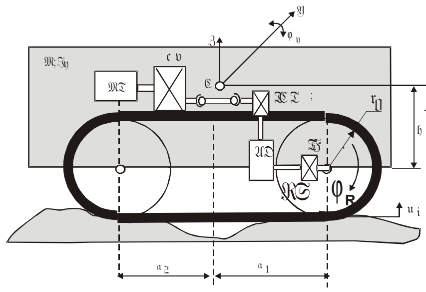

Fig. 4. The physical model of the equipment on caterpillar
MT- thermic; CV –gear box; TC-card an transmission; TCD- transmission station; AD- clutch of direction; TF- final transmission; RS - the driving wheel caterpillar.

The real model of the equipment on caterpillars is suggested in the fig. 4. It was achieved in the next hypotheses: it was not taken in the considering the coupler, respectively the principal and the direction, considering that these are coupled during of analysated motion; The dynamic load is realize in equal mode on left caterpillar and the right and consequently the equipment is considered as having one caterpillar on which is seat the equipment. The traction system acts the wheels of caterpillar situated in the front of the equipment. It is taken in considering the characteristics of rigidity of the movement mechanism on the vertical direction of rolling path. The aim of the model is emphasized dynamic influential dislevelments rolling path on transmission as the previous case.

2. 2. 1. Modelling of movement of the equipment in conditions of adherence

As it is figure 5, the movement of the equipment with variable speed v is possible in the conditions of the adherence with the rolling path, under the act of traction force (F_0) from the caterpillars, to structure of the equipment.

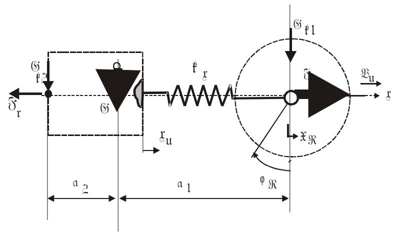

Fig. 5. Dynamic model for equipments on caterpillars

It is remarked the fact that induction of the motor moment at structure of the equipment as the traction force is achieved by the structure of caterpillar demonstrated through this rigidity on the movement direction. Fall-backing on structure of the equipment from figure 4, which represent with active and resistance forces is obtained the dynamic model from figure 5.

The differential equation what describes the dynamic process of movement of the equipment, is:

$$M\ddot{X}_R + k_x\left(X_R - X_U\right) = F_0 - F_r; \qquad (11)$$

with the signification term described previously.

2. 2. 3. The final mathematical model

For whole the equipment contenting: source of energy (thermic engine) transmission mechanical the driving wheel (caterpillar) rolling path structure equipment, the dynamic model can be considered a system a mechanic with three degrees of freedom, presented the in fig. 6.

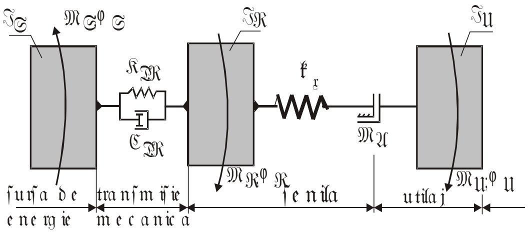

Fig. 6. The dynamic model of the equipment on caterpillars

The equations describe the dynamic process of energetic transfer from the thermic engine to structure of the equipment by the transmission of the mechanic and wheels of caterpillar, in conditions of adherence ($M_R < M_A$) are:

$$\begin{cases} J_s\ddot{\varphi}_s + r^2 c_{TR}\left(\dot{\varphi}_s - i_{TR}\dot{\varphi}_R\right) + \\ + r^2 k_{TR}\left(\varphi_s - i_{TR}\varphi_R\right) = M_s; \\ J_R\ddot{\varphi}_R + k_\varphi\left(\varphi_R - \varphi_U\right) - i_{TR} r^2 c_{TR}\left(\dot{\varphi}_s - i_{TR}\dot{\varphi}_R\right) - \\ - i_{TR} r^2 k_{TR}\left(\varphi_s - i_{TR}\varphi_R\right) = -M_R; \\ J_T\ddot{\varphi}_U - k_\varphi\left(\varphi_R - \varphi_U\right) = -M_U; \end{cases} \qquad (12)$$

Terms from (12) have the signification described to the previous model, and have the expressions:

$$M_{R1} = G.f.r_0; \qquad (13)$$

$$M_{R2} = 2.[k_{z1}\left(z - u_S - a_1\varphi_y\right) + k_{z2}(z - u_D - a_2\varphi_y)].f.r_0; \qquad (14)$$

$$M_U = 0; \qquad (15)$$

On the relations of the moments (13), (14), is come back when is caused these expression the result from the movement across the dislevels run away.

If in the system reply the moment M_S with the correspondent relation obtained from characteristic regulator orders proper terms, results the mathematical model of dynamic behavior of the system STIM-S:

$$\begin{cases} J_S\ddot{\varphi}_S + c_{11}\dot{\varphi}_S + c_{12}\dot{\varphi}_R + k_{11}\varphi_S + \\ + k_{12}\varphi_R = \alpha_S; \\ J_R\ddot{\varphi}_R + c_{21}\dot{\varphi}_S + c_{22}\dot{\varphi}_R + k_{31}\varphi_S + \\ + k_{32}\varphi_R + k_{33}\varphi_U = -M_R; \\ J_U\ddot{\varphi}_U + k_{32}\varphi_R + k_{33}\varphi_U = -M_U; \end{cases} \qquad (16)$$

The unknown the model are: $\varphi_S, \varphi_R, \varphi_U$, and the parametric sizes ale of the model are: M_R and M_U, what represent the factors perturbation inducted the in the system of kinematic excitation rolling path.

3. Conclusions

The system of differential equations (10) represents the mathematical model which allows the analysis of dynamic behavior of rigged equipments with STIM with systems of run on tire for different conditions of movement modeling through the state of rolling path, respectively through mathematical structure of the moment M_R.

The system of differential equations (16) he represents the mathematical model which allows analysis of dynamic behavior of rigged equipments with STIM with systems of run on caterpillars for different conditions of movement modeling through the state of rolling path, respectively through mathematical structure of the moment M_R.

Numerical modeling of the two systems permitted as for certain values what characterize certain technological equipments to traced diagrams of dynamic behavior, just as presented in the figures nr. 7, 8, 9, 10, 11, 12, for the case which in the moment of excitation, to his wheel caterpillars the equipment, is harmonic form represent through functions with the expression: [3].

$$M_R = M_{RS}[1 + \psi \sin(\omega_0 t - \Gamma)] \qquad (17)$$

Mathematical modeling gives us answers of traction system STIM at kinematic excitation rolling path (17):

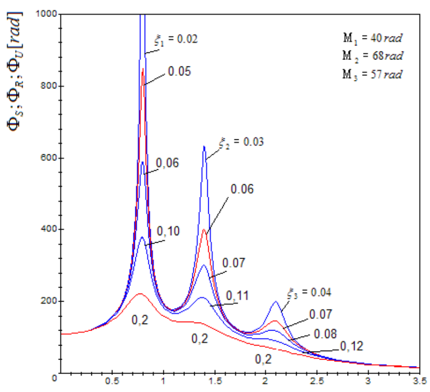

Fig. 7

Fig.7

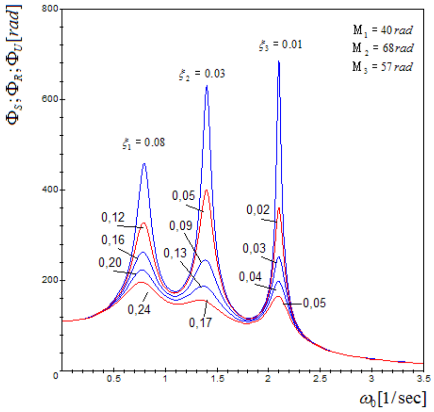

Fig. 8

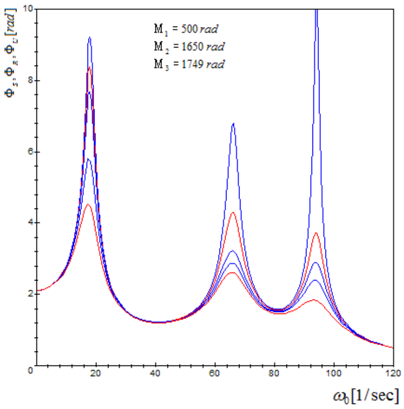

Fig.9

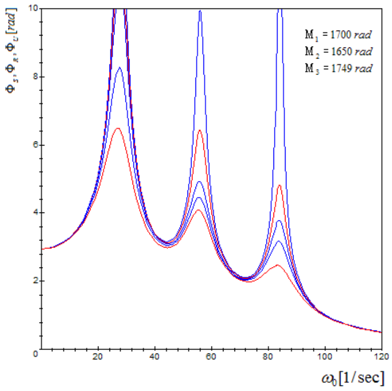

Fig.10

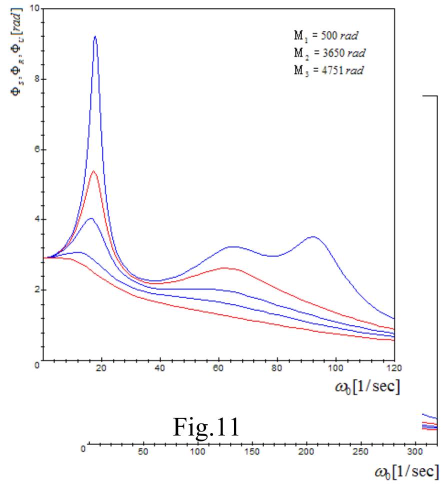

Fig.11

Fig.12

- The diagrams represent the variation of instantaneous angle at: the axle engine thermic Φ_S; its wheel caterpillars the equipment Φ_R or the angular movement of the equipment Φ_U, for different angular speeds ω_0 to the organ of movement of the equipment (wheel or caterpillar), excited kinematic of the run away through resistant formal another couples 17.
- Another conclusions about the complicated specific of the problems will be presented by the author in another papers.
- Must remarked that are presented in the paper under theoretical appearance

phenomena of resonance to twist inducted of the bed bearer about the draft systems ale of technological self-propelled equipments to equip with STIM.

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