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THE DYNAMIC MODELLING OF THE MECHANICAL SYSTEMS. CALCULUS OF THE EQUIVALENT MASS AND EQUIVALENT MASS INERTIA

Lecturer Dr.Eng. Carmen Nicoleta DEBELEAC
Assoc.Prof.Dr.Eng. Nicuşor DRĂGAN
The Research Center of Machines, Mechanic and Technological Equipments
"Dunarea de Jos" University of Galati

ABSTRACT

This study presents the phases and the final result of the physical and mathematical models' elaboration of mechanical systems with elastical and rigid elements taking into consideration the equivalent mass and inertia. The result and final considerations have a real utility in fast and operational calculus of the natural frequencies of this kind of mechanical models with more degree of freedom.

1. Introduction

Reducing the mechanical characteristics of a mechanical equipment or technical system consists in the calculation of the equivalent values of their structural elements (masses, inertia, elasticities, dampings) and for acting forces and moments.

2.The kinetic energy of mechanical systems

To reduce masses and mass inertia of a mechanical system means to calculate the equivalent values for these parameters, so that the kinetic energy is the same. Thus, for the calculus of the masses and inertia of equivalent system, the adequate method use the principle of kinetic energy equalization between the real system and reduced system.

It considers a certain mechanical system which is composed from rigid bodies with movements of translation and rotation with fixed axis; this hypothesis don't particularizes the problem because, it's demonstrable that, in the rigid body general movement, the kinetic energy can be written as a sum of the translation energy and fixed axis rotation energy.

Assuming that the system is composed from n elements with translation movements (with the masses m_i $i=\overline{1,n}$ and velocities v_i $i=\overline{1,n}$) and p elements with fixed axis rotation movements (with the mass inertia J_k $k=\overline{1,n}$ and angular speeds ω_k $k=\overline{1,n}$), the total kinetic energy of it has the formula:

$$E=\sum_{i=1}^{n}\frac{1}{2}m_i v_i^2+\sum_{k=1}^{p}\frac{1}{2}J_k\omega_k^2 \qquad (1)$$

The kinetic energy of the equivalent system with one single mass with a translation movement is

$$E=\frac{1}{2}m_r v^2\ , \qquad (2)$$

where v is the velocity of the system

$m_r \equiv m_{eqv}$ - the reduced / equivalent mass.

Similarly, if the equivalent system has one single mass with a fixed axis of rotation movement, the kinetic energy is

$$E=\frac{1}{2}J_r\omega^2\ , \qquad (3)$$

where ω is the angular speed

$J_r \equiv J_{eqv}$ - the reduced / equivalent mass inertia.

If it has to reduce the entire system to a reduced system with one translation mass, the equivalent mass formula is to be obtained from the relations *(1)* and *(2)* as follows:

$$m_r \equiv m_{eqv} = \sum_{i=1}^{n} m_i\left(\frac{v_i}{v}\right)^2 + \sum_{k=1}^{p} J_k\left(\frac{\omega_k}{v}\right)^2 \qquad (4)$$

But, if it has to reduce the system to an equivalent one with fixed axis of rotation, the reduced mass inertia can be obtained from *(1)* and *(3)* as follows:

$$J_{eqv} \equiv J_r = \sum_{i=1}^{n} m_i\left(\frac{v_i}{\omega}\right)^2 + \sum_{k=1}^{p} J_k\left(\frac{\omega_k}{\omega}\right)^2 \qquad (5)$$

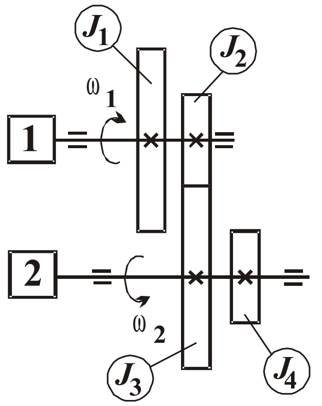

Fig. 1 The calculus model of one step gear

3.The equivalent mass inertia for the one step gearing transmissions

To understand the method to equivalate the mass inertia, it considers the transmission with one step gears from *figure 1*, where $J_i \;\; i=\overline{1,4}$ are the inertia of the gears and $\omega_i \;\; i=\overline{1,2}$ are the angular speed of the shafts. The reducing of the inertia can be made at the two shafts.

The equivalent inertia of the gear wheel **3** at the shaft no. **1** is

$$J_{3eqv} \equiv J_{3r} = J_3\left(\frac{\omega_2}{\omega_1}\right)^2 = J_3\frac{1}{i_{12}^2} = J_3 i_{21}^2 \;, \qquad (6)$$

where $$i_{12} = \frac{\omega_1}{\omega_2} \qquad (7)$$

and $$i_{21} = \frac{\omega_2}{\omega_1} \qquad (8)$$

are the speed ratio of the transmission (i_{12} from shaft **1** to shaft **2**, i_{21} from shafts **2** to **1**).

If the reducing has to be made at the shaft no. **2**, the equivalent mass inertia of the gear wheel **2** has a calculus formula like this:

$$J_{2eqv} \equiv J_{2r} = J_2\left(\frac{\omega_1}{\omega_2}\right)^2 = J_4\frac{1}{i_{21}^2} = J_4 i_{12}^2 \qquad (9)$$

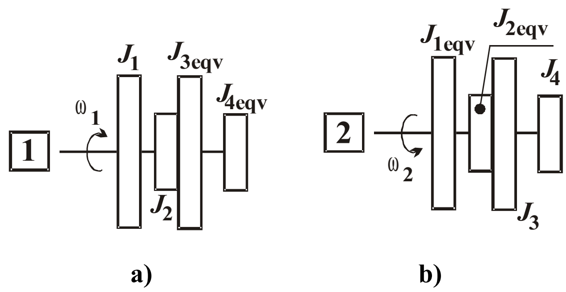

Fig. 2 The calculus diagrams for the inertia
a)the equivalent inertia on shaft no. **1**
b)the equivalent inertia on shaft no. **2**

4.Physical and mathematical models of the mechanical transmissions with gears and rigid shafts

It's assuming that the mechanical system from *figure 2* is composed from rigid and no mass shafts and the gearing is ideal (mechanical efficiency is unitary). In this way, the wheel gears **1** and **2** have the same angular speed ω_1 and the wheel gears **3** and **4** have angular speed ω_2 both of them.

4.1.Reducing system on the shaft no. 1

The equivalent system reduced to the shaft no. **1** is shown on *figure 2 a)*. The mass inertia of the gearing is

$$J_{23}^{(1)} = J_2 + J_{3eqv} \;, \qquad (10)$$

where J_{3eqv} is done by the relation *(6)*.

The total mass inertia of the system on the shaft no. **1** is

$$J_{tot}^{(1)} = J_1 + J_{23}^{(1)} + J_{4eqv} \;, \qquad (11)$$

where J_{4eqv} is the reduced inertia of the wheel gear **4** to the shaft no. **1** like this:

$$J_{4eqv} \equiv J_{4r} = J_4\left(\frac{\omega_2}{\omega_1}\right)^2 = J_4\frac{1}{i_{12}^2} = J_4 i_{21}^2 \qquad (12)$$

4.2.Reducing system on the shaft no. 2

The equivalent system reduced to the shaft no. **2** is shown on *figure 2 b)*. The mass inertia of the gearing is

$$J_{23}^{(2)} = J_{2eqv} + J_3 \;, \qquad (13)$$

where J_{2eqv} is done by the relation *(9)*.

The total mass inertia of the system on the shaft no. **2** is

$$J_{tot}^{(2)} = J_4 + J_{23}^{(2)} + J_{1eqv} \;, \qquad (14)$$

where J_{1eqv} is the reduced inertia of the wheel gear **1** to the shaft no. **2** as follows:

$$J_{1eqv} \equiv J_{1r} = J_1\left(\frac{\omega_1}{\omega_2}\right)^2 = J_3\frac{1}{i_{21}^2} = J_3 i_{12}^2 \quad (15)$$

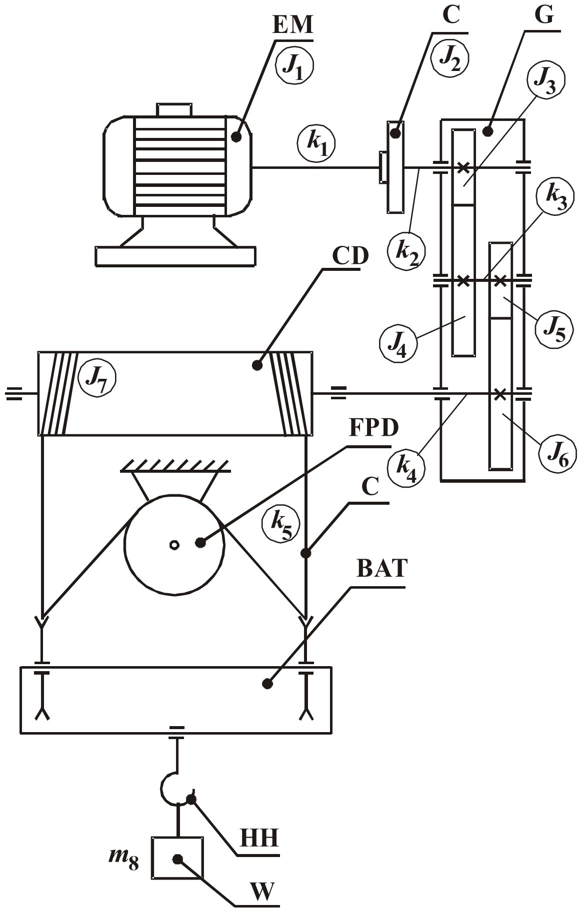

Fig. 3 The skeleton diagram for the sprocket winch of a bridge crane

EM-electromotor
C-coupling
G-gear reducer unit
CD-cable drum
C-cable
BAT-block-and-tackle
FPD-fixed pulley device
HH-hoisting hook
W-weight

5.Reducing the masses and mass inertia for complex mechanical systems

The equivalation of the masses and mass inertia for the mechanical system is mainly based on the general relations *(4)* and *(5)*. It's obvious that, the application of the two formula depends on the structure of each particular system.

For exemplification, it considers the mechanism of a bridge crane which diagram skeleton is shown in the *figure 3*. The complete mechanical system of the driving's crane is more complex but, the main elements are those from the diagram.

The mechanical characteristics of the structural elements accordingly to the skeleton diagram are as follows

►**masses and inertia**

J_1 - inertia of the electromotor's rotor

J_2 - inertia of the coupling (clutching)

J_3, J_4, J_5 and J_6 - wheel gears' inertia of the reducing gear

J_7 - inertia of the cable drum

m_8 - total hanging mass (lifting weight mass plus hoisting hook mass plus block-and-tackle mass)

►**modulus of elasticity**

k_1 - elasticity of the driving shaft

k_2 - elasticity of the primary shaft of the gear reducer unit

k_3 - elasticity of the intermediate shaft of the gear reducer unit

k_4 - elasticity of the driven shaft of the gear reducer unit (the winding barrel of winch)

k_5 - elasticity of the cable

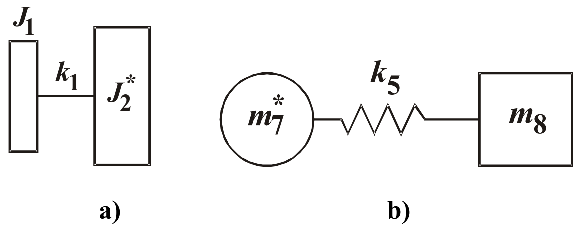

Fig. 4 The simplified calculus diagrams
a)assuming that the driving shaft is the most elastic
b)assuming that the cable is the most elastic element

Depending on the scope of the reducing operation and the desired accuracy of the calculus, it may make some simplifying hypothesis. For example, if the most elastic element is the driving shaft of the electromotor (meaning k_1 is the smaller) and the equivalation has to be made on the electromotor shaft, the simplified diagram for calculus is that from *figure 4 a)*. In this case, the equivalent mass inertia J_2^* is as follows

$$J_2^* = J_2 + \sum_{i=3}^{8} J_{ieqv} \ , \quad (16)$$

where J_{ieqv} $i = \overline{3,8}$ are the equivalent mass inertia (depending on real mass characteristics and on speeds/velocities of system elements).

Presuming that the system equivalation is made at the hoisting weight and the cable is the most elastic element, it can simplify the calculus diagram like that one from *figure 4 b)*. In this case, the equivalent mass m_7^* is as follows

$$m_7^* = \sum_{i=1}^{7} m_{ieqv} \ , \qquad (17)$$

where m_{ieqv} $i = \overline{1,7}$ are the equivalent mass.

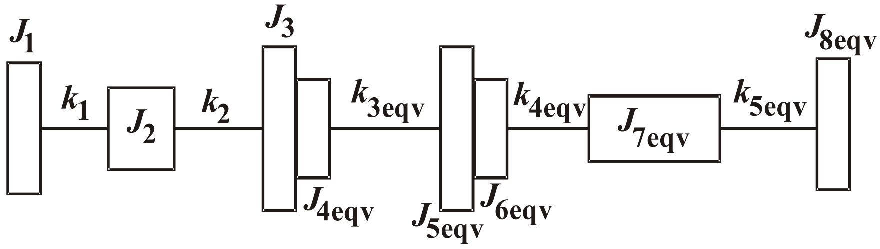

Fig. 5 The calculus diagram for the equivalent inertia and flexibilities on the electromotor shaft

6. Conclusions

The operations of mass characteristics' reduction of a mechanical system are mainly made on the kinetic energy equalization principle. Basically, this consists in the construction of an equivalent calculus model so that its total kinetic energy is equal with the kinetic energy of the real system one.

The equivalation of the mass characteristics of a mechanical system may be made for simplified models if promptness (short calculus time) is the priority; there are the cases of simplified models from *figure 6*. But, if the dynamic analysis needs a high precision grade and the time of modeling and calculus is not an explicit request, it have to considerate all the elements of the system and their mass or elastic characteristic. Thus, if the reducing of the system from *figure 3* has to be made on the electromotor driving shaft, a more exactly calculus model is that from *figure 5*, where also the equivalent elasticities have been taken into consideration. The *figure 6* shows the calculus model of the system from *figure 3* with the reduced characteristics (masses, inertia, elasticities) on the cable drum shaft.

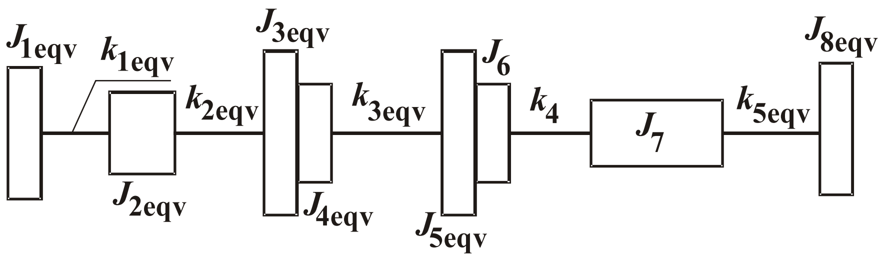

Fig. 6 The calculus diagram for the equivalent inertia and flexibilities on the cable drum shaft

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