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ALL - PURPOSE LOW FREQUENCY NUMERICAL FILTER

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ABSTRACT

A real signal is a complex system with mixed components. In addition to needed components appear unusable components with different origin, namely noise. The clearance or the reduction of the unwanted components is realized on more ways, i.e. with analogical filter devices built-up on the signal capture or processing channel, or with numerical filters. This last way can be applied only on already digitized signal and it is necessary a high-speed calculator. The work presents an all-purpose numerical filter for low frequency domain, namely sound or mechanical vibration, with an excellent noise suppression or signal spectrum improvement.

1. Introduction

A really signal contains more needed and unusable components from different frequency, amplitude and phase. In order to make a numerical processing of the signal this must be firstly digitized, respectively sampled from analogical form similar to analytical continuous function on one certain time domain into a numerical discontinuous two-dimensional function with values only at certain moments. If the signal contains components up to highest frequency f_h or if we consider that the signal is sufficient known up to the component frequency f_h, the sampling rate must be chosen under the consideration of the "sampling theorem", id est. the temporal length δt between any two successive sampled values (named sampling period) must be littler as the value give from [1]

$$\delta t \le \frac{1}{2f_h} \tag{1}$$

or, for a maximal accessible sampling frequency f_s , $f_s = 1/\delta t$, the highest analyzed frequency has the value

$$f_h = \frac{1}{\delta t} \le \frac{f_s}{2} \tag{2}$$

The formula (2) shows that the highest frequency of the processed signal is limited up to half of the sampling frequency. If on present-day PC the maximal sampling rate (sampling frequency) is not greater as 40---100 kHz, the highest analyzed frequency must be under 20---50 kHz.

2. The numerical filter

The filter is based on the Fourier direct and inverse transformation. The Fourier direct transformation gives the spectral components of any signal on a certain frequency domain. In our case the limit of the frequency domain is between minimal 0 and maximal f_h concordantly with the sampling theorem. A real signal $s(t)$ is transformed into another function $F(j\omega)$ with the formula [1]

$$F(j\omega) = \int_{-\infty}^{\infty} s(t) e^{-j\omega t} dt \tag{3}$$

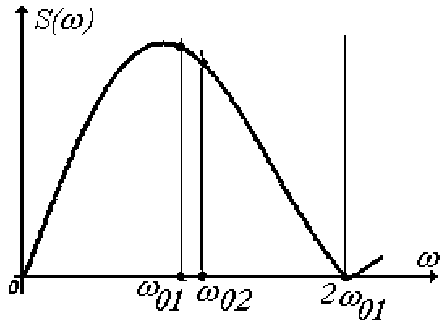
with ω=pulsation, $\omega = 2\pi f$, $j = \sqrt{-1}$. Because the signal has a finite temporal length between t_1 , t_2 and it is null out of this interval, the expression (3) can be written

$$\begin{aligned} F(j\omega) &= \int_{t_1}^{t_2} s(t) e^{-j\omega t} dt = \int_{t_1}^{t_2} s(t) cos(\omega t) dt - \\ &- j \int_{t_1}^{t_2} s(t) sin(\omega t) dt = Re(j\omega) + j\, Im(j\omega) \end{aligned} \tag{4}$$

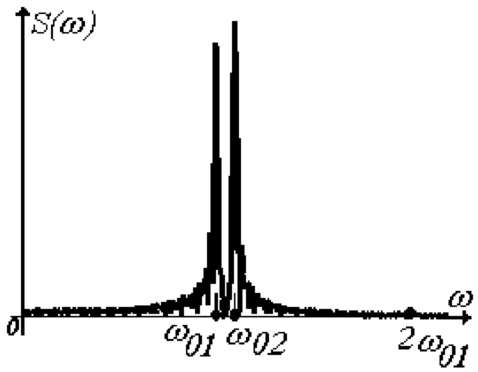
namely a complex function with two parts: really and imaginary. The amplitude of this complex function, designated with $S(j\omega)$ and calculated with

$$S(j\omega)=\sqrt{Re^2(j\omega)+Im^2(j\omega)} \qquad (5)$$

is named spectral function and represents the component frequencies of the signal $s(t)$. If the temporally length of the signal $s(t)$ is very short the spectral function contain spectral bands with large width [1], [5], induced only from the shortness of the signal, long signals give spectral functions with small band width. Spectrally, a short signal is not equal with a long signal with same components, fig. 1a) and 1b). Therefore each signal must be processed only itself.



a) The length $t_2 - t_1 = 1/50\ sec.$



b) The length $t_2 - t_1 = 1$ sec.

Fig. 1. Two spectral functions of the signal $s(t)=sin(\omega_{01}t)+sin(\omega_{02}t)$, $\omega_{01}=100\pi, \omega_{02}=110\pi$.

The inverse Fourier transformation give the original signal from his direct transformation [1]:

$$s(t)=\frac{1}{\pi}\int_0^{\infty} Re\left[F(j\omega)e^{j\omega t}\right]d\omega \qquad (6)$$

Because the highest pulsation in not infinite, the signal is given from

$$s(t)=\frac{1}{\pi}\int_0^{\pi f_s}\left[Re(j\omega)cos(\omega t)-Im(j\omega)sin(\omega t)\right]d\omega \qquad (7)$$

If under the integral from (7) we multiply the expression with a factor dependent from pulsation ω, namely $\Phi(\omega)$, the regenerated signal $s'(t)$ results influenced from this factor. This factor, $\Phi(\omega)$, modifies the aspect of the signal $s(t)$ and can be named "filter function". We obtain hereby a processed signal $s'(t)$

$$s'(t)=\frac{1}{\pi}\int_0^{\pi f_s}\begin{bmatrix}Re(j\omega)cos(\omega t)- \\ -Im(j\omega)sin(\omega t)\end{bmatrix}\Phi(\omega)d\omega \qquad (8)$$

3. Examples

We choose the signal with 8 spectral components described from

$$s(t)=\sum_{i=1}^{8} A_i\ sin[(8+2i)t] \qquad (9)$$

with $A_1=1$, $A_2=1.2$, $A_3=2$, $A_4=1.6$, $A_5=0.6$, $A_6=1.4$, $A_7=0.8$, $A_8=0.4$, presented in figure 2.

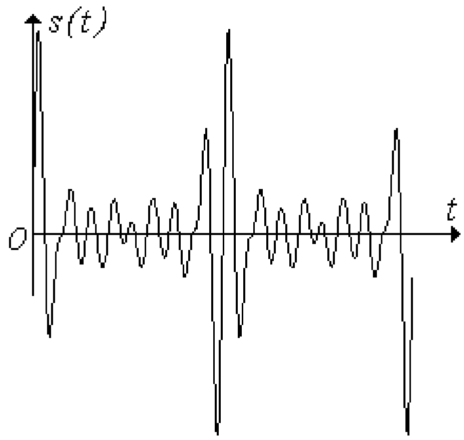


Fig. 2. The form of the signal (9)

We consider in fig. 2 that this signal has a temporal length between $t_1=0$ and $t_2=10\frac{2\pi}{10}$, respectively then time greater as the greatest period from (9). The spectral function determined with (5) on $\omega\in[0,30]$ is presented in figure 3. All 8 components appear very visible in figure 3 but, because the signal has a short length, are clear visible the large bandwidth on each component and other lateral components induced from the shortness of the signal.

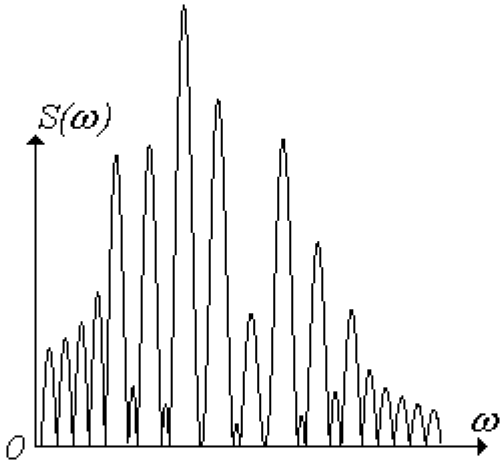


Fig. 3. The spectrum of the signal (9)

The regenerated signal *s'(t)* with (8) is made under consideration of the filter function

$$\Phi(\omega)=\begin{cases} 0 & for \quad \omega \in [15, 21] \\ 1 & for \quad \omega \notin [15, 21] \end{cases} \qquad (10)$$

and has the form presented in figure 4

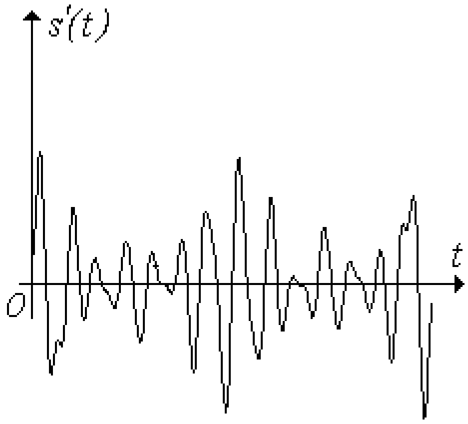


Fig. 4. The regenerated signal from (9) with the filter function (10)

His spectral function, figure 5, shows clear that the filter function act on the signal as a "band stop" function between ω=15 and 21 rad/sec.

If the filter function act on the signal as a "band pass" function

$$\Phi(\omega)=\begin{cases} 1 & for \quad \omega \in [13, 18] \\ 0 & for \quad \omega \notin [13, 18] \end{cases} \qquad (11)$$

results another signal, visible in figure 6 with spectral function presented in figure 7.

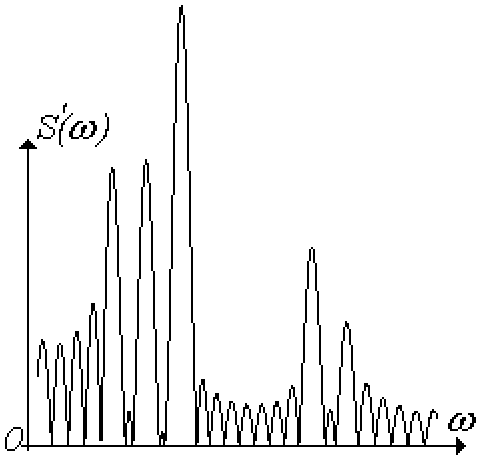


Fig. 5. Spectral function of the signal from figure 4

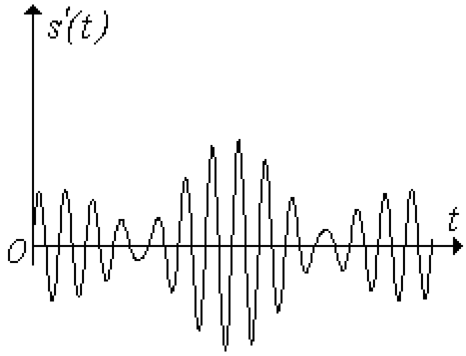


Fig. 6. The regenerated signal from (9) with the filter function (11)

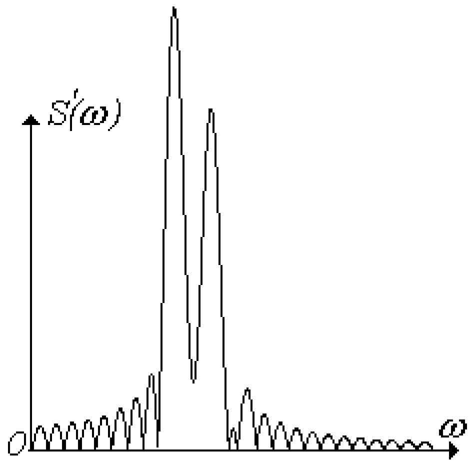


Fig. 7. The spectrum of the signal from figure (6)

The filter function can be build for more applications. At last example we chose a linear filter function with the expression

$$\Phi(\omega) = \frac{\omega}{24} \qquad (12)$$

applied at the signal

$$s(t) = \sum_{i=1}^{8} 1 \cdot \sin[(8+2i)t] \qquad (13)$$

The figure 8 presents the form, the figure 9 the spectrum of the signal (13).

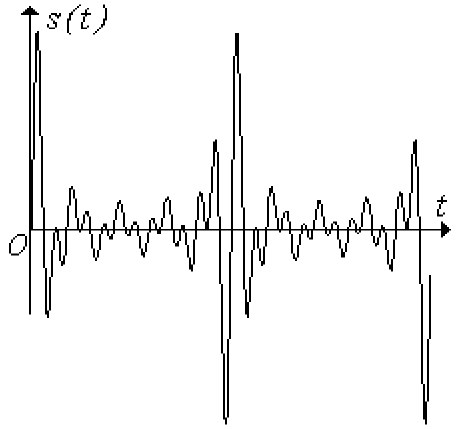


Fig. 8. The form of the signal (13)

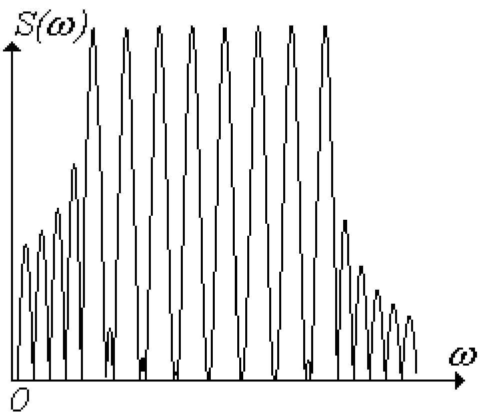


Fig. 9. The spectrum of the signal (13)

The regenerated signal from (13) with (12) has the form given in figure 10 and spectrum in figure 11.

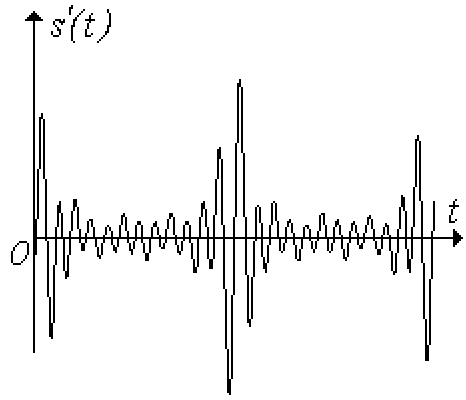


Fig. 10. The regenerated signal from (13) with the filter function (12)

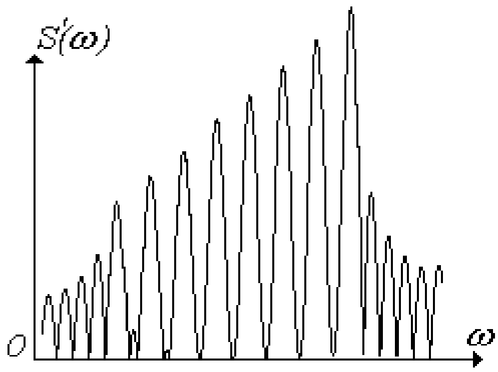


Fig. 11. The spectrum of the signal from figure 8

4. Conclusions

The processing of the numerical signal with numerical "filter functions" offers large possibilities in the domain of the low frequency signal analysis. Relatively to analogical filter, the numerical filter has following properties:

-it acts on the signal very efficient, precise on the desired domain and exactly in desired manner; analogical filter has invariably a transitory band at begin and end of his domain of the frequencies, mostly clear visible at "band pass" or "band stop" filter; this is the greatest advantage of the numerical filter, namely not a transitory band and a mathematical precise implement of the filter function;

-the signal must be firstly digitized and stored, then converted into complex function with formula (4) and finally processed with a desired filter function; these transformations need long processing time and can't offer immediate the processed signal same as analogical filter; this is the greatest disadvantage of the upper proposed numerical filter.

The results are obtained on a Pentium IV PC programmed from the author in DELPHI 6.

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