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ASPECTS REGARDING THE VIBRATIONS OF THE CONTACTS OF A COMPRESSED AIR SWITCH

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ABSTRACT

The paper studies the vibrations of the contacts of a compressed air switch, by considering two models. Time variation of the elongation is presented, as well as spectral representations determined by means of Fourier analysis. The results are in agreement and make up a useful tool for the design and the operation of this type of switches.

1.Introduction

Turning on an electric contact implies the collision between the movable element and the fixed one. According to the construction of the device, this collision may be more or less elastic.

The vibration of the contact, induced by the repulsion of the movable element by the fixed one, as a consequence of the collision process, is harmful to the electric device, since it leads to the modification of the contact electric resistance and, finally, after the separation of the elements, to the appearance of the electric arc .

The study of the above mentioned vibratory phenomenon is performed in the present paper by using two models of the dynamic system constituted by the contacts of a switch.

The first model studies the described phenomenology, while the second one admits the existence of a perturbation force with a known law, the response of the system being determined by means of Fourier analysis.

2.Study models

The contacts of a compressed air switch are schematically presented in Figure 1, where the following notations were used :

m - mass of the movable element;

F_P - force acting on the movable element;

M - mass of the fixed element, which is supported by an elastic spring;

K - elasticity constant of the spring.

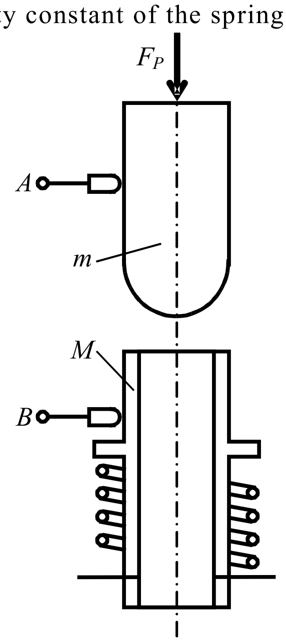


Fig. 1. Scheme of a compressed air switch

The study of the vibrations is performed by means of two models, which will be presented in the following.

2.1. First model of the switch

The first model of the switch (Fig. 2) is the simplest one and it assumes that the movable contact is a mass , m acted upon by the force F_P, which induces to it a uniformly accelerated motion. This motion ends when the movable contact touches the fixed one, which has the mass M, and which is supported by the elastic spring with the elasticity constant K.

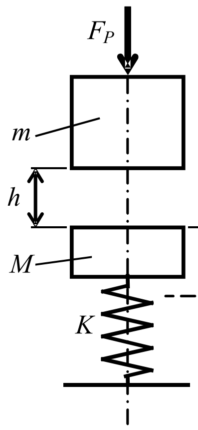


Fig. 2. First model of the switch

The vibration starts at the moment of the contact of the two elements and it is described by the differential equation

$$(M+m)\ddot{x}(t)+Kx(t)=F_P\ . \qquad (0)$$

When the collision takes place $(t=0)$, the velocity of the assembly of mass $M+m$ is considered equal to the velocity of the body m, accelerated on the distance h:

By considering the origin of the axis x at the upper limit of the contact, when the spring in not deformed, the following initial conditions result:

$$t=0:\begin{cases} x(0)=-\delta=-\dfrac{F_P}{K} \\ \dot{x}(0)=\sqrt{2ha}=\sqrt{\dfrac{2hF_P}{m}}, \end{cases} \qquad (0)$$

where δ is the static deformation of the spring.

The differential equation (0) is linear and non-homogeneous and its general solution has the form

$$x=x_h+x_P, \qquad (0)$$

where x_h is the general solution of the corresponding homogeneous equation, while x_P is a particular solution of the non-homogeneous one:

$$x_h=C_1\sin\omega t+C_2\cos\omega t,\quad x_P=\frac{F_P}{K}. \qquad (0)$$

In the previous formula

$$\omega=\sqrt{\frac{K}{M+m}}, \qquad (0)$$

while C_1 and C_2 are integration constants, which are determined from the initial conditions (0).

After determining and replacing the integration constants, solution (0) becomes

$$x=\delta-\delta\cos\omega t+\sqrt{\frac{2(M+m)}{m}\cdot\frac{hF_P}{K}}\sin\omega t\ . \qquad (0)$$

By assuming that mass M is small compared to m,

$$\frac{M+m}{m}\approx 1 \qquad (0)$$

and by using the known formula

$$C_1\sin\omega t+C_2\cos\omega t=A\sin(\omega t-\varphi),$$

relation (0) takes the form

$$x=\delta+\sqrt{\delta(2h+\delta)}\sin(\omega t-\varphi). \qquad (0)$$

The time variation of the elongation x is presented in Figure 3, which shows that the dynamic arrow differs from the static one and varies between the limits

$$\begin{cases} x_{\max}=\delta+\sqrt{\delta(2h+\delta)} \\ x_{\min}=\delta-\sqrt{\delta(2h+\delta)}. \end{cases} \qquad (0)$$

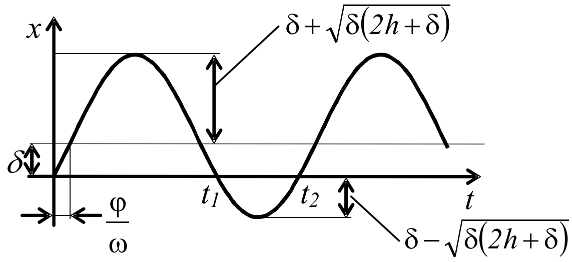


Fig. 3. Time variation of the elongation

It can be remarked that, during the time interval $[t_1,t_2]$, the arrow is negative, which corresponds to the tendency of the movable contact to detach from the fixed one.

If the detachment does not take place, i.e. if the contacts move with the same acceleration, the pressure force F_a acting upon the two elements is positive $(F_a\geq 0)$.

In order to determine F_a, variable $\ddot{x}$ is eliminated from the equations of motion of the contacts:

$$m\ddot{x} = F_P - F_a, \quad M\ddot{x} = F_a - Kx. \qquad (0)$$

It follows:

$$F_a = \frac{MF_P + mKx}{m + M}. \qquad (0)$$

The condition $F_a \geq 0$, which can be written as

$$x \leq -\frac{M}{m} \cdot \frac{F_P}{K}, \qquad (0)$$

must be fulfilled for the least favorable value of the elongation x, i.e. for $x_{\min}$. By using the second relation (0), this condition becomes

$$K \leq \frac{MF_P}{2m^2 h}(M + 2m). \qquad (0)$$

Inequality (0) is fulfilled for a force F_P large enough (greater than a minimum value). A smaller mass m will also lead to the elimination of the vibration.

It should be noted that relation (0) does not taken into account the friction, which contributes to the diminishing of the vibrations, neither does it describe the increase of the electric resistance, due to the decrease of the pressure force, which leads to excessive heating of the contacts.

2.2.Second model of the switch

The study of the vibrations of a compressed air switch can be performed by means of a second model (Fig. 4), which will be presented in the following .

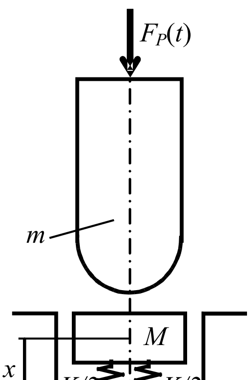


Fig. 4. Second model of the switch

For this second model, the Fourier spectrum of the amplitude for different values of the constant K will be determined, corresponding to a chosen perturbation force (Fig. 5),

$$F_P(t) = \begin{cases} F_0 \cos\dfrac{\pi t}{2\tau} & \text{for} \quad -1 \leq \dfrac{t}{\tau} \leq 1 \\ 0 & \text{for} \quad 1 < \dfrac{t}{\tau} \leq 7. \end{cases} \qquad (0)$$

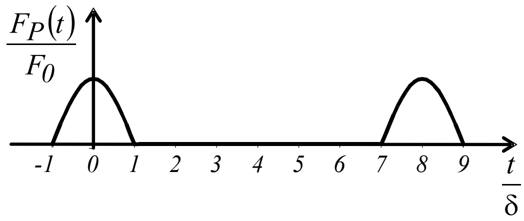


Fig. 5. Variation law of the perturbation force

From the variation law of the perturbation force, the period and the perturbation circular frequency, respectively, can be determined:

$$\begin{cases} T = 8\tau \\ \Omega = \dfrac{2\pi}{T} = \dfrac{\pi}{4\tau} = \dfrac{\pi}{4}\omega_0, \end{cases} \qquad (0)$$

where

$$\omega_0 = \frac{1}{\tau}. \qquad (0)$$

The coefficients of the Fourier series expansion , which allow the analysis of the vibration $x(t)$, are

$$a_0 = \frac{1}{T}\int_0^T F_P(t)dt = \frac{2}{T}\int_0^\tau F_0 \cos\left(\frac{\pi t}{2\tau}\right)dt = \frac{F_0}{2\pi}, \qquad (0)$$

$$a_2 = \frac{4F_0}{8\tau}\left[\frac{t}{2} + \frac{2\tau}{4\pi}\sin\frac{\pi t}{\tau}\right]\Bigg|_0^\tau = \frac{F_0}{4}, \qquad (0)$$

$$\begin{cases} a_k = \dfrac{2}{T}\displaystyle\int_0^T F_P(t)\cos(k\Omega t)dt = \\ = \dfrac{4F_0}{8\tau}\displaystyle\int_0^\tau \cos\left(\frac{\pi t}{2\tau}\right)\cos\left(\frac{k\pi}{4\tau}\right)dt = \\ = \dfrac{F_0}{\pi} \cdot \dfrac{4}{4 - k^2}\cos\left(\dfrac{k\pi}{4}\right) \quad (k = 1,3,4,...), \end{cases} \qquad (0)$$

$$\{b_k = 0 \quad (k = 0,1,2,...). \qquad (0)$$

The last relation takes place because the function $F_P(t)$ is even,

$$F_P(-t) = F_P(t). \qquad (0)$$

The numerical values of the coefficients a_k are listed in , while the corresponding amplitude spectrum is represented in Figure 6.

Table 1

k	a_k / F_0
0	0.159
1	0.300
2	0.250
3	0.180
4	0.106
5	0.043
6	0.000
7	-0.020

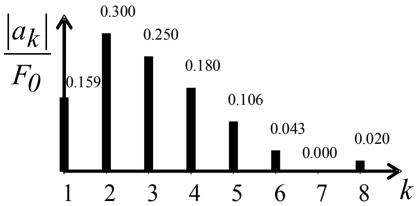


Fig. 6. Amplitude spectrum of the perturbation force

A discrete spectral representation is obtained, specific to the deterministic vibrations.

The study can be continued by determining the amplitude spectrum of the elongation x of the fixed contact.

The equation of motion of the fixed contact is

$$M\ddot{x} + Kx = F_P(t), \qquad (0)$$

where

$$F_P(t) = \sum_{k=0}^{\infty} a_k \cos(k\Omega t). \qquad (0)$$

The solution of equation (0) is

$$x(t) = \frac{1}{K}\left[\sum_{k=0}^{\infty} \frac{a_k}{1-(k\eta)^2}\cos(k\Omega t)\right], \qquad (0)$$

where

$$\omega = \sqrt{\frac{K}{M}}, \quad \eta = \frac{\Omega}{\omega}. \qquad (0)$$

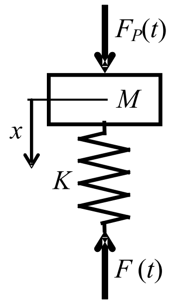


Fig. 7. Model for the determination of the response of the fixed contact

The response of the fixed contact, determined by means of the model in Figure 7, without considering the weight, is

$$F(t) = K \cdot x(t) = \sum_{k=0}^{\infty} \frac{a_k}{1-(k\eta)^2}\cos(k\Omega t). \qquad (0)$$

The results of the analytical calculus of $F(t)$ are presented Table 2, for two values of the elasticity constant K.

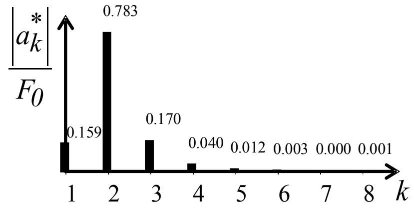
Table 2

Case	I	II
K	$K = M\omega_0^2$	$K = 0.25M\omega_0^2$
$\omega = \sqrt{\frac{K}{M}}$	ω_0	$0.5\omega_0$
$\eta = \frac{\Omega}{\omega}$	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$\frac{a_k}{1-(k\eta)^2}$	$\frac{a_k}{1-\left(\frac{k\pi}{4}\right)^2} = a_k^*$	$\frac{a_k}{1-\left(\frac{k\pi}{2}\right)^2} = a_k^{**}$
$F(t)$ (18)	$\sum_{k=0}^{\infty} a_k^* \cos(k\Omega t)$	$\sum_{k=0}^{\infty} a_k^{**} \cos(k\Omega t)$

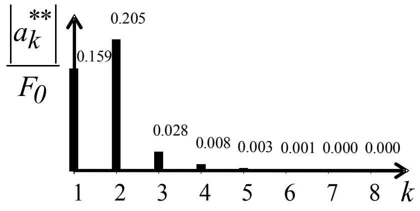
Numerical values obtained for the two situations are presented in Table 3, while the corresponding amplitude representations in Figure 8.

Table 3

k	a_k^* / F_0	a_k^{**} / F_0
0	0.159	0.159
1	0.783	-0.205
2	-0.170	-0.028
3	-0.040	-0.008
4	-0.012	-0.003
5	-0.003	-0.001
6	0.000	0.000
7	0.001	0.000



a) $K = M\omega_0^2$



b) $K = 0.25M\omega_0^2$

Fig. 8. Amplitude spectra of the response of the fixed contact

By comparing the amplitude spectra in Figure 8, it follows that the forces acting upon the fixed contact decrease considerably when the elasticity constant decreases with *25*%. This remark leads to the conclusion that the isolation of the vibration can be achieved by a convenient choice of the elastic spring.

3.Results and conclusions

The paper analyses, by means of two models, the vibration of the contacts of a compressed air switch.

The analytical and the numerical results lead to the conclusion that avoidance of the vibratory phenomenon between the contacts of the switch depends on the value of the elasticity constant.

The response of the system, determined in the two situations by different methods, provides, by a relatively simple calculus, necessary information for the design of efficient switches of this type.

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