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THE ANALYSIS OF THE TORSIONAL SPRINGS' INERTIA INFLUENCE ON THE RESONANCE CHARACTERISTIC OF THE ELASTICAL MECHANICAL SYSTEMS

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ABSTRACT

The study presents the phases and the final result of the physical and mathematical models' elaboration of elastic mechanical systems with one degree of freedom taking into consideration the distributed mass of the twist beam spring. The result and final considerations have a real utility in fast and operational calculus of the natural frequency of this kind of mechanical models, pointing out the influence of the distributed mass of twist beam spring for resonance characteristics.

1. Introduction

The mechanical systems with one freedom degree of rotational movement has minimum one inertial element and one elastic twist element, usually being modelled like in *figure1*.

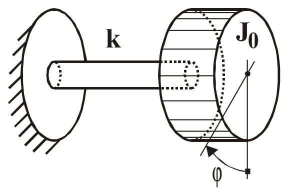

Fig. 1 The model of the mechanical system with one degree of freedom - rotational movement

The resonance pulsation for the calculus model from *figure 2 a)* is

$$p_0 = \sqrt{\frac{k_\varphi}{J_0}} \tag{1}$$

and the natural frequency

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k_\varphi}{J_0}} \ , \tag{2}$$

where J_0 is the reduced inertia of the system (neglecting the torsional beam spring mass) and k_φ is the torsional coefficient of stiffness.

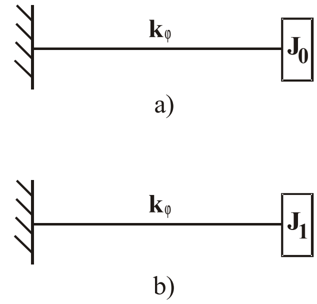

Fig. 2 The calculus models for resonant frequency of the mechanical system
a) *the system neglecting the spring mass*
b) *the system with total equivalent mass*

2.Physical model of the system

The *figure 3* shows the calculus diagrams for a simple mechanical system with rotational movement and a torsional bend spring with circular section.

It presumes that all the dimensional, elastic and inertial characteristics of the system are known:

● for the axial mounted flywheel: the mass and the axial inertia J_0

● for the torsional beam: the mass and the axial inertia J, the length l, the polar inertia I_p, the torsional coefficient of stiffness

Without to make a particular approach

of this problem, it considers that the fixed beam is made from homogeneous material and the sectional area is a constant circular shape.

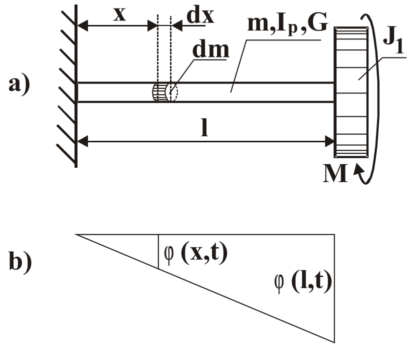

Fig. 3 The calculus diagrams for resonat equivalent inertia

3.Mathematical model

The *figure 3* shows the diagram for calculus of the equivalent inertia of the system flywheel-torsional beam.

According to Hooke law, if G is the modulus of torsion, on the acting of a torsional moment M_t, the rotation displacements φ are:

■for the section x

$$\varphi(x,t) = \frac{M}{GI_p}x \tag{3}$$

■for the end of the beam

$$\varphi(l,t) = \frac{M}{GI_p}l \tag{4}$$

From *(3)* and *(4)*, the relation between the two rotation displacements is:

$$\varphi(x,t) = \frac{x}{l}\varphi(l,t) \tag{5}$$

Deriving the relation *(5)*, it obtains the relation between the spin velocities as follows:

$$\dot{\varphi}(x,t) = \frac{x}{l}\dot{\varphi}(l,t) \tag{6}$$

Presuming that the section of the beam is full circular where r is the radius of it, the hatchet element dm from the section x has the inertia

$$dJ = \frac{dm}{2}r^2 = \frac{1}{2}\frac{m}{l}dx \cdot r^2 \tag{7}$$

If the torsional beam is a pipe with $2R$ outside diameter and $2r$ inside diameter, the inertia of the infinitesimal mass dm is

$$dJ = \frac{1}{2}\frac{m}{l}dx\left(R^2 - r^2\right) , \tag{8}$$

and the kinetic energy is as follows:

$$dE = \frac{1}{2}dJ[\dot{\varphi}(x,t)]^2 \tag{9}$$

Taking into consideration the relations *(6)* and *(7)*, the kinetic energy becomes:

$$dE = \frac{1}{2}\frac{mr^2}{2l^3}[\dot{\varphi}(l,t)]^2 x^2 dx \tag{10}$$

The total kinetic energy of the beam is to get by integrating the relation (10), the final expression of it being

$$E = \int dE = \frac{1}{2}\frac{J}{3}[\dot{\varphi}(l,t)]^2 , \tag{11}$$

where $J = \frac{1}{2}mr^2$ is the inertia of the entire torsional beam (full section).

Because the kinetic energy of the beam calculated with the rotational velocity of the end section has the expression

$$E = \frac{1}{2}J_r[\dot{\varphi}(l,t)]^2 , \tag{12}$$

the equivalent inertia of the beam in this section is obtained from *(11)* and *(12)* as follows:

$$J_{eqv} = \frac{1}{6}mr^2 = \frac{J}{3} \tag{13}$$

Consequently, the calculus model is that shown in the *figure 2 b)* and the total equivalent inertia of the system is

$$J_1 = J_{eqv} + J_0 = \frac{J}{3} + J_0 \tag{14}$$

4. Conclusions

The physical and the calculus model of the mechanical system with a torsion elastic beam and a flywheel on the end of it make evident the influence of the distributed mass of the beam on the resonance behavior. This influence is bigger how much more the beam mass is bigger and the end mass is smaller.

In any case, the equivalent inertia mass of the system is bigger than the inertia of the flywheel, therefore the value of resonance pulsation is decreasing. The modifications of the calculus inertia and of the resonance frequency are very important because can lead to major modifications of the dynamic behavior of the system, especially for transient regimes (starting, stopping).

5. References

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