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THE ANALYSIS OF THE AXIAL SPRINGS' WEIGHT INFLUENCE ON THE RESONANCE CHARACTERISTIC OF THE ELASTICAL MECHANICAL SYSTEMS

Conf. dr. ing. Nicuşor DRĂGAN
Universitatea "Dunărea de Jos" din Galaţi

ABSTRACT

This study presents the phases and the final result of the physical and mathematical models' elaboration of elastic mechanical systems with one degree of freedom taking into consideration the distributed mass of the axial spring. The result and final considerations have a real utility in fast and operational calculus of the natural frequencies of this kind of mechanical models, pointing out the influence of the mass of axial spring beam for resonance characteristics.

1. Introduction

The mechanical systems with one freedom degree has minimum one mass element and one elastic element, usually being modelled like in *figure no. 1 a)*. Every system with one freedom degree, no matter how many elements (mass elements, elastic/spring elements, neglecting damping elements) can be reduced to the model of the *figure 1 a)*.

The resonance pulsation for the reduced model is

$$p = \sqrt{\frac{k}{m_1}} \qquad (1)$$

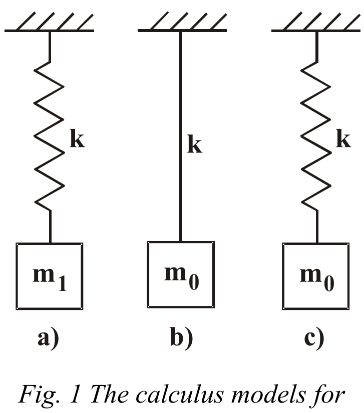

Fig. 1 The calculus models for resonant frequency of the mechanical system
a) *the system neglecting the spring mass*
b) *the system with total equivalent mass*
c) *the equivalent system with coil spring*

and the natural frequency

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m_1}} \;, \qquad (2)$$

where m_1 is the reduced mass of the system (neglecting the spring mass) and k is the coefficient of stiffness.

2.Physical model of the system

The *figure no. 2* shows the calculus diagrams for a simple mechanical system with an axial spring bend and a single concentrated mass in translation movement of vibration. For simplification, it consider an homogeneous material for the beam and a constant transversal section, therefore the value of the produce EA is constant, where E is the elastic modulus and

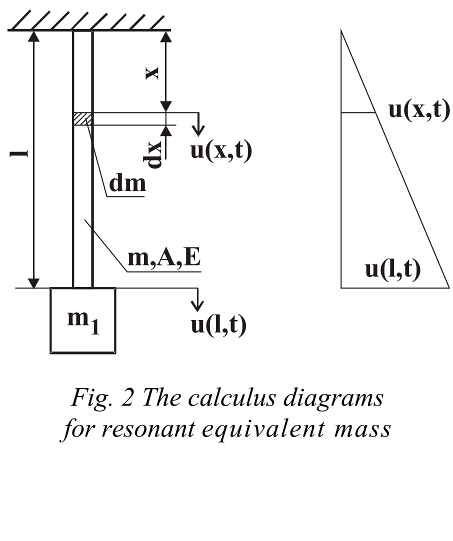

Fig. 2 The calculus diagrams for resonant equivalent mass

A is the area of the section.

The linear specific mass of the beam is

$$\rho_l = \frac{m}{l} \ , \tag{3}$$

the mass of the infinitesimal hatching element being

$$dm = \rho_l dx = \frac{m}{l} dx \tag{4}$$

The study of the reducing mass of the spring can be done in two different hypothesis for the calculus of the static deformation of the beam:

1)taking into consideration only the weight of mass m_1;

2)taking into consideration the total weight of mass m_1 and the mass m of the beam.

3.Mathematical model of the system

3.1.Statical elongation done by the mass m_1

According to Hooke law, the elastic elongation of the section located on the distance x from the clamping is

$$u(x,t) = \frac{G_1}{EA} x \tag{5}$$

and the displacement of the beam's end is

$$u(l,t) = \frac{G_1}{EA} l \tag{6}$$

The relation between the two displacement is

$$u(x,t) = \frac{x}{l} u(l,t) \ , \tag{7}$$

through derivation obtaining the relation between the velocities:

$$\dot{u}(x,t) = \frac{x}{l} \dot{u}(l,t) \tag{8}$$

The kinetic energy of the element dm located on the distance x is

$$dE = \frac{1}{2} dm \cdot [\dot{u}(x,t)]^2 = \frac{1}{2} \frac{m}{l} \left(\frac{x}{l} \right)^2 [\dot{u}(l,t)]^2 dx \tag{9}$$

The energy for the entire beam is obtained by integration

$$E = \int dE = \int_0^l \frac{1}{2} \frac{m}{l} \left(\frac{x}{l} \right)^2 [\dot{u}(l,t)]^2 dx \tag{10}$$

After the calculus of the integral *(10)*, it results the total kinetic energy of the beam as follows:

$$E = \frac{1}{2} \frac{m}{l^3} [\dot{u}(l,t)]^2 \left. \frac{x^3}{3} \right|_0^l = \frac{1}{2} \frac{m}{3} [\dot{u}(l,t)]^2 \tag{11}$$

Since the general relation of the kinetic energy of the reduced mass is

$$E = \frac{1}{2} m_r v^2 \ , \tag{12}$$

where m_r is the reduced mass and v is the velocity of the section where the reducing calculus is to be done, through identification of the inertial therms from *(11)* and *(12)* it obtains the expression of the reduced/equivalent mass of the beam on the end of it:

$$m_r \equiv m_{eqv} = \frac{m}{3} \tag{13}$$

The total kinetic energy of the mechanical system is obtaining by summing the energy of the beam and the energy of the mass element m_1

$$E = E_{beam} + E_{m_1} = \frac{1}{2} m_0 [\dot{u}(l,t)]^2 \ , \tag{14}$$

where m_0 is the total reduced/equivalent mass of the system in the section end of the beam (where is located the mass m_1); the diagram of the equivalent system is shown in *figure 1 b)*.

Through the identification of the terms of the kinetic energies, it obtains expression of the total reduced/equivalent mass:

$$m_0 = \frac{m}{3} + m_1 \tag{15}$$

Analysing the relation *(15)*, it may take the next preliminary conclusions:

-one third of the beam mass is taking into consideration for the calculus of the total kinetic energy of the system;

-the total mass of the system is bigger than the mass m_1, therefore the value of resonance pulsation is decreasing;

-the equivalent calculus diagram is shown in *figure 1 c)*, the resonance pulsation and the natural frequency having the following expressions:

$$p_{eqv} = \sqrt{\frac{k}{m_1}} \tag{16}$$

$$f_{eqv} = \frac{1}{2\pi} \sqrt{\frac{k}{m_1}} \tag{17}$$

3.2.Statical elongation done by the mass m_1 and the beam mass m

Taking into consideration that the mass of the beam is homogeneous distributed, the linear specific weight of it is

$$q = \frac{mg}{l} \ , \tag{18}$$

where q is the specific weight and g is the gravity acceleration.

The total force which produces the

displacement of the section situated on the distance x from the clamping is done by summing the weight m_1 and the weight of the part of the beam located under this section. The expression of the total force is

$$F = q(l - x) + G_1 = g\left[m\left(1 - \frac{x}{l}\right) + m_1\right] \qquad (19)$$

According to Hooke law, for section x it may write

$$\frac{F}{A} = E\frac{u(x,t)}{x} , \qquad (20)$$

from where the displacement of the section is

$$u(x,t) = \frac{F}{EA}x \qquad (21)$$

Taking into consideration the expression *(19)* of the force F, the differential of the displacement done by *(21)* is

$$du(x,t) = \frac{F}{EA}dx = \frac{g}{EA}\left[(m_1 + m) - m\frac{x}{l}\right]dx \qquad (22)$$

Integrating the relation *(22)*, it can obtains the expression of the displacement of any section of the beam. Thus, the displacement of the section x is as follows:

$$u(x,t) = \int_0^x du(x,t) = \frac{g}{EA}\int_0^x\left[(m_1 + m) - m\frac{x}{l}\right]dx \qquad (23)$$

Consequently, the displacement of the beam's end has the expression:

$$u(l,t) = \int_0^l du(x,t) = \frac{g}{EA}\int_0^l\left[(m_1 + m) - m\frac{x}{l}\right]dx \qquad (24)$$

By calculating the integrales (23) and (24), it obtains the final expressions for the displacements of the section x and of the end of the beam as follows:

$$u(x,t) = \frac{gx}{EA}\left[(m_1 + m) - m\frac{x}{2l}\right] \qquad (25)$$

$$u(l,t) = \frac{gl}{EA}\left(m_1 + \frac{m}{2}\right) \qquad (26)$$

From *(25)* and *(26)* it can write the relation between the two displacements

$$u(x,t) = \lambda\, u(l,t) , \qquad (27)$$

where the λ is a constant parameter as follows:

$$\lambda = \frac{x}{l}\frac{2m_1 + m\left(2 - \frac{x}{l}\right)}{2m_1 + m} \qquad (28)$$

Deriving *(27)*, it obtains the relation betweens the velocities of the section x and the end of the beam:

$$\dot{u}(x,t) = \lambda\, \dot{u}(x,t) \qquad (29)$$

The kinetic energy of the element dm is

$$dE = \frac{1}{2}dm[\dot{u}(x,t)]^2 = \frac{1}{2}\frac{m}{l}dx \cdot \lambda^2[\dot{u}(l,t)]^2 \qquad (30)$$

or

$$dE = \frac{1}{2}\frac{m}{l}\left[\frac{x}{l}\frac{2m_1 + m\left(2 - \frac{x}{l}\right)}{2m_1 + m}\right]^2[\dot{u}(l,t)]^2 dx \qquad (31)$$

The total kinetic energy of the beam is calculating by integrating the infinitesimal energy done by the relation *(31)*

$$E = \int dE = \frac{1}{2}A \cdot I \cdot [\dot{u}(l,t)]^2 , \qquad (32)$$

where:

$$A = \frac{m}{l^3(2m_1 + m)^2} = const. \qquad (33)$$

$$I = \int_0^l\left[2(m_1 + m)x - \frac{m}{l}x^2\right]^2 dx \qquad (34)$$

After the calculus, the expression of the integrale *(34)* is

$$I = \frac{l^3}{15}\left(20m_1^2 + 25m_1m + 8m^2\right) , \qquad (35)$$

thus the total kinetic energy of the beam is

$$E = \frac{1}{2}\frac{20m_1^2 + 25m_1m + 8m^2}{15(2m_1 + m)^2}m \cdot [\dot{u}(l,t)]^2 \qquad (36)$$

Because the reduced mass of the beam is to be done in the end of it, the expression of the kinetic energy it have to be as follows:

$$E = \frac{1}{2}m_r[\dot{u}(l,t)]^2 \qquad (37)$$

Through the identification of the terms of the relations *(36)* and *(37)*, the equivalent/reduced mass of the beam in the end of it has the expression

$$m_r \equiv m_{eqv} = \frac{20\mu^2 + 25\mu + 8}{15(2\mu + 1)^2}m = \beta(\mu) \cdot m , \qquad (38)$$

where the dimensionless parameters μ and β are the follows:

$$\mu = \frac{m_1}{m} \qquad (39)$$

$$\beta(\mu) = \frac{20\mu^2 + 25\mu + 8}{15(2\mu + 1)^2} \qquad (40)$$

Analysing the relations *(38)-(40)*, it may take the next partial conclusions:

-only one fraction of the beam mass is taking into consideration for the calculus of the total kinetic energy of the system;

-the fraction of beam mass which participates into the calculus of the equivalent mass depends on the mass m_1; this fraction is described by the parameter β which is bigger if the mass of the end of the beam is smaller and inversly
-the maximum influence of the beam mass is reached in absence of the mass m_1:

$$\beta_{max} = \beta(0) = \frac{8}{15} \Rightarrow \ m_{r\,max} = \frac{8}{15} m \quad (41)$$

-the influence of the beam mass is decreased if the value of mass m_1 is very big:

$$\beta_{min} = \beta(\infty) = \frac{1}{3} \Rightarrow \ m_{r\,min} = \frac{1}{3} m \quad (42)$$

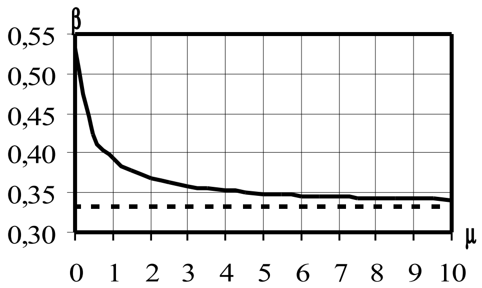

Fig. 3 The variation of the **β** *parameter depending on "dimensionless mass"* **μ**

-the variation of the parameter β (which measures influence mass of the beam on the equivalent mass) depending on the "dimensionless mass" μ) is shown in the *figure no. 3*

4. Conclusions

The physical and the calculus model of the mechanical system with an axial elastic beam and a concentrated mass on the end of it make evident the influence of the distributed mass of the spring on the resonance behavior. This influence is bigger how much more the beam mass is bigger and the end mass is smaller.

In any case, the total mass of the system is bigger than the concentrated mass m_1, therefore the value of resonance pulsation is decreasing. The modification of the calculus mass and of the resonance frequency is very important especially for the machines with the resonant function regime.

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