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ON THE LINEAR ELASTIC DEFORMATIONS FOR PLANE CURVED BEAMS IN THEIR PLAN LOADED, USING THE FINITE ELEMENT METHOD

Conf. dr.ing. Petru DUMITRACHE
Prep. ing. Aurora POTIRNICHE
UNIVERSITATEA “DUNAREA DE JOS” DIN GALATI

ABSTRACT

The study want to make a comparison between tho calculus methods of the linear elastic deformations for plane curved beams in their plan loaded. The results obtained with the finite element analysis have some important advantages against the results obtained with the classic calculus of resistence' strength..

1. INTRODUCTION

The determination of the tensions and deformations who is developing in the structural elements, loaded with strengths resulted by the functional role achievement, are representing a primal problem.

The classic methods of the material's strength have limits, because it is considered some simplify hypothesis, which are necessary to the analytical integration of the equations resulted after the mathematical calculus.

The simplify hypothesis take in discussion, in principal, the schemes of the structural elements geometry, the schemes of the materials behaviour, of the strengths.

Thence, the precision of the obtained results depend of the accepted level of the schemes.

The finite elements method permit a treating of a superior quality of this problem because can be realize models of the structural elements very approached by reality. It's important to mention that it can be modeled geometries and boundary conditions extremely complicated and material's behaviours having at base experimental determinations.

In the case of structural elements, which are on the limit of the hypothesis which stay at base of the calculus relations determination in the materials strength, it is necessary a verification of the precision level offered by these calculus relations.

So, the finite elements method is a very good performance work instrument.

This study propose the verification of the results precision obtained by the classical method applied to the calculus of the curved beams deformations (in arc of circle). This beams are solicited by the coplanar loads, in the plan of elastic line.

It will be considered three cases:

- a curved beam in arc of circle, in the limits of calculus hypothesis
- a curved beam in arc of circle, which accomplish the calculus hypothesis, but is to the limit of these
- a curved beam in arc of circle, which not accomplish the calculus hypothesis, but is to the limit of these.

In every case will be applied the classic calculus method and the finite elements method and will be made an analysis of the results precision.

2. THE THEORETICAL CONSIDERATIONS

In the case of rigidity calculus for a plane curved beam with rectangular transverse section is important the value of $\frac{\rho}{h}$ ratio as a limit of calculus hypothesis, where ρ is the curvature radius of the beam axis and h is the section side in the curvature plan of beam.

So, for $\frac{\rho}{h} = 3$ is the case to the limit between the big curvature beam and the small curvature beam, in the calculus hypothesis limits.

For $\frac{\rho}{h} \geq 3$ the beam has a small curvature, in this case the calculus hypothesis are accomplished, but the case is to the limit.

For $\frac{\rho}{h} \leq 3$ the beam has a big curvature, the calculus hypothesis are not accomplished, but the case is to the limit of these.

It will be determinated the displacements which appear in a plane beam with small curvature, loaded with strength in the beam plan, like in the next drawing.

It suppose that for all the beam transverse sections one of the central inertia axis is also in the beam plan.

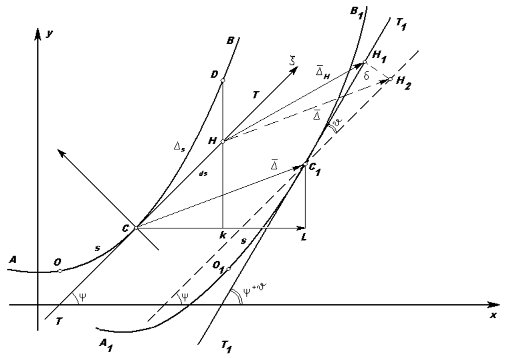

Figure 1 The displacements which appear in a plane beam with small curvature, loaded with strength in the beam plan.

The mathematical calculus guide to the next relations:

$$\frac{du}{dy} = -\vartheta \tag{1}$$

$$\frac{dv}{dx} = \vartheta \tag{2}$$

$$\frac{d\vartheta}{ds} = \frac{M}{EI} \tag{3}$$

These relations describe the differential equations of the elastic line for the plan curved beam with a small curvature, stressed to bending in his plan.

To deduce the differential equations of the elastic lines, it was adopted the following sign conventions for the variables of the equations.

1) It will be considered positive the bending moment M if, after the bending, the beam's curvature is growing up, therefore

$$\frac{1}{\rho} > \frac{1}{\rho_0}$$

2) It will be considered positive the angular displacements □, if, after the beam deformation, the tangent at beam's axis is rotated from X axis to Y axis.

3) It will be considered positive the linear displacements □si □, if they coincide with the positive orientations of the coresponding coordinates axes.

To integrate the differential equations (1), (2), (3) it's necessary to express all the terms contained by those equations, function by a certain parameter. For the beams with a circle arc form axis, it's profitable to consider that the central angle □ is a principal parameter. This parameter give the position of a certain transverse section.

In this case the angular displacement value $\vartheta(\varphi)$ is :

$$\vartheta(\varphi) = \vartheta_0 + \frac{1}{EI}\int_{\varphi_0}^{\varphi} M(\varphi)\, ds(\varphi), \tag{4}$$

and the linear displacement values $u(\varphi)$ and $v(\varphi)$ are :

$$u(\varphi) = u_0 - \int_{\varphi_0}^{\varphi} \vartheta(\varphi)\, \mathrm{d}y(\varphi) \tag{5}$$

$$v(\varphi) = v_0 + \int_{\varphi_0}^{\varphi} \vartheta(\varphi)\, \mathrm{d}x(\varphi) \tag{6}$$

3. THE CASE STUDY

It will be considered a circular beam with the radius r, cuted like in the figure 2.

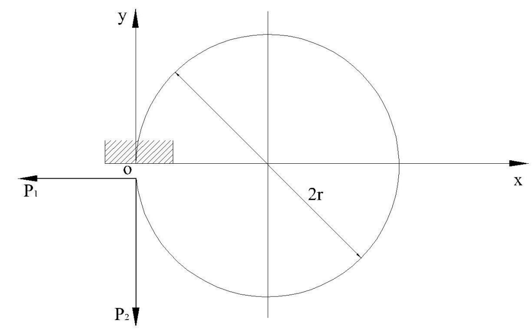

Figure 2 The circular beam which is used like a model for case studies

One of the section faces is fixed and the other one is loaded with two perpendicular strength P_1 and P_2, which are in the axis plan of the circular beam

It will be study the elastic line of the circular beam in the reminded three cases, using the classic calculus of material's strength as well as the finite element method.

In the case of classic calculus, the results was obtained with the Mathcad program.

Case	Results, mm		Eror, %
	The classical formula	FEA result	
$\rho = 120\ mm$, $h = 24\ mm$, $b = 8\ mm$, $\frac{\rho}{h} = 5$	$u = -0.028$	$u = -0.028$	0
	$v = -0.084$	$v = 0.084$	0
	$\delta = 0.089$	$\delta = 0.089$	0
$\rho = 74\ mm$, $h = 24\ mm$, $b = 8\ mm$, $\frac{\rho}{h} = 3.08$	$u = -0.007$	$u = 0.008$	12.50
	$v = -0.020$	$v = 0.022$	9.1
	$\delta = 0.021$	$\delta = 0.023$	8.70
$\rho = 60\ mm$, $h = 24\ mm$, $b = 8\ mm$, $\frac{\rho}{h} = 2.5$	$u = -0.004$	$u = -0.006$	33.33
	$v = -0.011$	$v = -0.070$	84.29
	$\delta = 0.012$	$\delta = 0.070$	82,85

In all the three cases was considered a circular beam with a transverse section 8×24 ($b \times h$) and loaded with unitar strengths. The difference between the cases is the size of curvature radius, which give the different ratio $\frac{\rho}{h}$.

The obtained results after the study of the three cases are synthetized into the table 1 (attach to this study).

We present also the result obtained with the finite element method in the second case, when the calculus hypothesis are accomplish, but is to the limit. We consider this case the most suggestive of the obtained results.

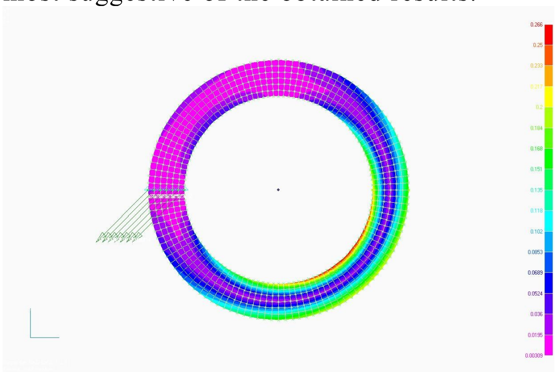

Figure 3 The model-only after the analysis, with finite element method resolved

The model presented, analyzed with the finite element method has, in the right side a scale of stresses, calculated with the von Mises theory of the resistance' s strength.

4. CONCLUSIONS

From this study can we obtain some important conclusions :

- the classic calculus is limited by the simplify hypothesis using, because, whithout these, it can not be resolved the differential equations which result from the mathematical model.
- owing to simplify hypothesis using , the obtained results on classic calculus base are taked off of reality.
- the most important advantage of the finite element method : this method not necesitate calculus simplify hypothesis and can solve ny mathematical model, however complicated is this.

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