THE ANNALS OF “DUNAREA DE JOS” UNIVERSITY OF GALATI
FASCICLE XIV MECHANICAL ENGINEERING, ISSN 1224 – 5615
2006

# COMPUTATIONAL DYNAMICS OF THE ELASTIC ANTIVIBRATIONAL SYSTEMS WITH NON-LINEAR STIFFNESS

Prof.assist. Eng. Silviu-Marian NASTAC
University “Dunarea de Jos” of Galati
Research Centre for Mechanics of the Machines and Technological Equipments

## ABSTRACT

*In this paper the author present a set of mathematical models, usable to simulate and analyse the static and dynamic behaviour aspects, both for the passive anti-vibrational and anti-seismical systems, and for all the machines and equipments that have movements on the unarranged roads, in the length time of a technological process. For all the considered models the author have write the characteristic system of movement equations and present the numerical simulation results, with taking into account the real equipments input data. The author was maded an analysis about the influences of the non-linear type stiffness on the dynamic behaviour of the antivibrational and antiseismical passive isolation equipments, with the impact on the dynamic performances of these protection systems. Also, it was analyse and evaluate the static and dynamic characteristics of these systems because it is very important to protect both the human resources, and the equipments, against vibratory pollution or seismical waves.*

## 1. Introduction

The problematics of the antivibrational and antiseismical isolation for the sensitives machineries and equipments supposes the presence of the specialised systems for reduce or eliminate the transmission phenomenon of the vibrations produced by the certain source, to the machinery or equipment with the necessary continous operating state, regardless to the environment conditions. Taking into account the operating way of the isolation ssytems, these could be divided into the next classes:

- ⇨ *passive systems* - the functional parameters that global characterised the isolation capacity, are imposed only by the stiffness and dissipative characteristics of the elastic components of the isolation systems
- ⇨ *active (adaptive) systems* - these systems contained, beside on the effective isolation components, the complex sub-system for aquisition, processing and adjustment the elastic and dissipative characteristics of the isolators; entire the components aquire continously the specific parameters of the vital equipments and make the necessary adjustments, on the real time, with the aim of the reducing the vibration transferability factor

The actual researches for finding optimal anti-seismic an anti-vibrational protection solutions are straightening to the few areas of activity: the first area of research is the base isolation of buildings or theirs parts, against the vibrations and earthquake actions. The second area of activity, in anti-seismic protection domain, is the isolation of vital equipments of public buildings endowment, against the seismic waves or disturbing vibrational movements.

All the systems achieved and used for passive anti-seismic and antivibrational protection of vital equipments, until present, are characterized both through the elastic performances according to obtain necesary natural frecquency (lowest values) on direction of dynamic degrees of freedom, and through the dissipative performancies so that the energy damping should substantial decrease the shock impact on the seismical wave direction.

In essence, antiseismic systems are based on suitable ensembling of individual elements or sub-systems, with proper elastic and damping characteristics, so that at standard dynamic loads spectral composition should be avoid the resonance dangerous phenomenon.

One way of characterize this systems is the isolation degree. In this work the author treat the problem of seismic isolation for vital equipments from public building endowment, using a new and innovator anti-seismic leaning system. One of the proposed scope of this entire research is obtaining the isolation degree values over the 90%.

In this paper are treats only one passive type of isolation system. The working principle of this systems are based on the elastic capability of the rubber elastic elements to assume the exciting loads energy and transforming it into the potential energy. In figure 1 it is presented a 3D model of the anti-vibrational system that will be analysed in this paper.

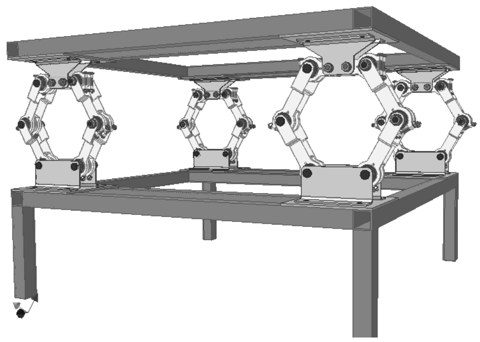


Figure 1. Antivibrational passive elastic system with special complex configuration

The nomenclature used in this paper are

*a* denote distances between the gravity center and the springs along the Ox axis;

*b* denote distance between the application point of the external force and the Oy axis;

*F1, F2* denote springy forces that replace the external *P* force;

*J* denote the moment of inertia of the isolated system;

$k_1$, $k_2$ denote stiffness constants of the springs;

*m* denote the mass of the isolated system;

*P* denote concentrated external force;

$\varphi$ denote angular degree of freedom for the isolated system;

$J_1$, $J_2$ denote the moments of inertia of the isolated system and the base (isolators included)

$m_1$, $m_2$ denote the masses of the isolated system and the base (isolators included)

$\varphi_{1,2}$ denote angular degree of freedom for the isolated system and the base

## 2. Computational Models

The first proposed model have two degree of freedom, namely translation between Oy axis and angular movement along the center of gravity into the Oxy plane. This physical model is presented in the figure 2.

For write the characteristic equations of this model that considered the next arrays: matrix of the inertial characteristics M, generalized movements vector q, the stiffness matrix K and external loads vector L, with the next forms

$$M = \begin{bmatrix} m & 0 \\ 0 & J \end{bmatrix} \tag{1}$$

$$q = \begin{bmatrix} y \\ \varphi \end{bmatrix} \tag{2}$$

$$K = \begin{bmatrix} (k1 + k2) & (k1 - k2)a \\ (k1 - k2)a & (k1 + k2)a^2 \end{bmatrix} \tag{3}$$

$$L = \begin{bmatrix} P \\ Pb \end{bmatrix} = P\begin{bmatrix} 1 \\ b \end{bmatrix} \tag{4}$$

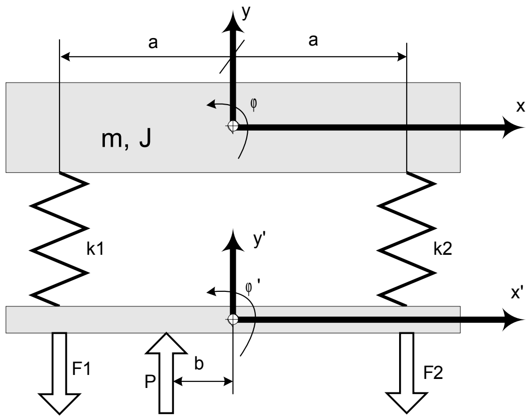



Figure 2. Physical model of isolation system - vs.I

With these notations, the mathematical model of the system presented in figure 1, is

$$M\ddot{q} + Kq = L \tag{5}$$

From eq. (3) it can be observed that both the system equations are coupled, and for separating them it must accomplish the next condition

$$k_1 = k_2 = k \tag{6}$$

The author considered that even all the elastic elements have the same type and operting caracteristic, in the real mode it could be differents, and of this motive, the link between the two variable $k_1$ and $k_2$ are

$$k_2 = k_1\alpha = k\alpha \tag{7}$$

where α is the geometric non-linearity coefficient.

Taking into account the design mode of the real system for vibration isolation and the working profile of these, the first proposed model could be modified through adoption of the next hypothesis: the numerical analysis will be maded with consider only the plane movements (in transverse plane of the system): displacements along the Oy axis and angular displacements along the center of gravity of masses m1 and m2. Thus, in figure 3 it is presented the physical model.

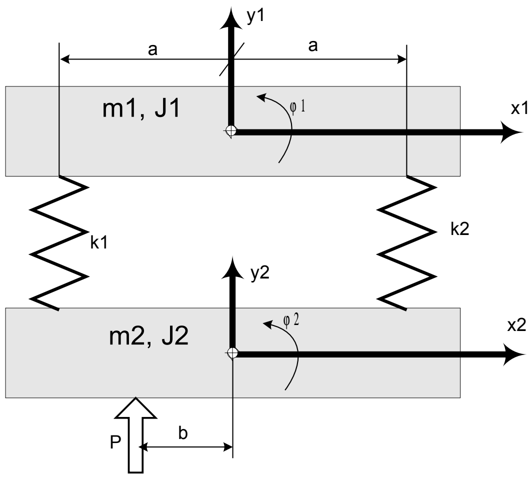



Figure 3. Physical model of isolation system - vs.II

For write the characteristic equations of this model that considered the next arrays: matrix of the inertial characteristics M, generalized movements vector q, the stiffness matrix K and external loads vector L, with the next forms

$$M = \begin{bmatrix} m1 & 0 & 0 & 0 \\ 0 & J1 & 0 & 0 \\ 0 & 0 & m2 & 0 \\ 0 & 0 & 0 & J2 \end{bmatrix} \tag{8}$$

$$q = \begin{bmatrix} y1 \\ \varphi 1 \\ y2 \\ \varphi 2 \end{bmatrix} \tag{9}$$

$$K = \begin{bmatrix} (k1+k2) & (k1-k2)a & -(k1+k2) & -(k1-k2)a \\ (k1-k2)a & (k1+k2)a^2 & -(k1-k2)a & -(k1+k2)a^2 \\ -(k1+k2) & -(k1-k2)a & (k1+k2) & (k1-k2)a \\ -(k1-k2)a & -(k1+k2)a^2 & (k1-k2)a & (k1+k2)a^2 \end{bmatrix} \tag{10}$$

$$L = \begin{bmatrix} 0 \\ 0 \\ P \\ Pb \end{bmatrix} = P\begin{bmatrix} 0 \\ 0 \\ 1 \\ b \end{bmatrix} \tag{11}$$

With this notations the mathematical model of the system presented in figure 3, are

$$M\,\ddot{q} + K\,q = L \tag{12}$$

From eq. (10) it can be observed that both the system equations are coupled, and for separating them it must accomplish the condition from eqn. (6).

The author considered that even all the elastic elements have the same type and operting caracteristic, in the real mode it could be differents, and of this motive, the link between the two variable $k_1$ and $k_2$ is shown by the eqn. (7)

The values $k_1$ and $k_2$ of the stiffness have constant values, but in real mode, the behaviour of the materials not respect the Hooke's law. With the other words, in real mode, the elastic element from system structure, have a non-linear characteristic. In this study, based on the experimental analysis for used elastic elements, the author consider that this characteristic could be write

$$k = k(t) = k' + k''\,x(t)^2 \tag{13}$$

or the elastic force expresion

$$Fe = Fe(t) = k'\,x(t) + k''\,x(t)^3 \tag{14}$$

For numerical simulation the dynamic behaviour of the proposed anti-vibrational system it was taken two case of external loads:

- first case is an harmonical force with constant magnitude and frequency

$$P(t) = A\sin(2\pi f\,t) \tag{15}$$

- second case is an seismical wave type, with time length about 3,5 sec

$$P(t) = 600e^{-1,3(t-1,7)^2}\left(0,3\sin(35t-59,5)+0,25\sin(130t)\right) \tag{16}$$

The diagram for the last type of the exciting force are presented in the figure 4.

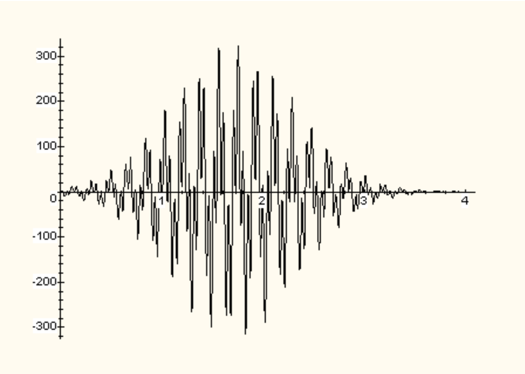


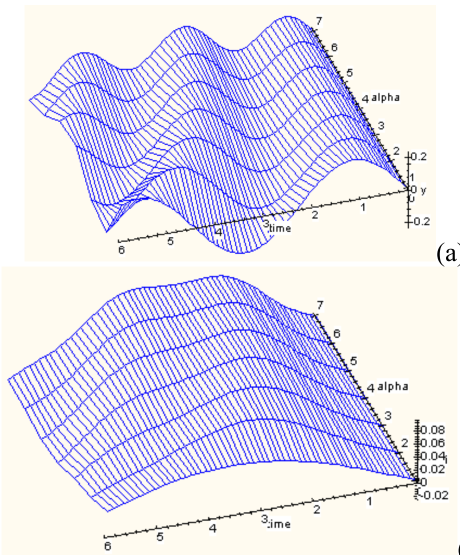

Figure 4. Temporal evolution of a simulated seismical wave

## 3. Computational Dynamics Results

For numerical simulation of the dynamic behaviour, the author using the next values for the constants that appear in the mathematical model (Table 1).

Table 1. Numerical values of the models

| constant name | value | units |
|---|---|---|
| *mass - m* | 100 | kg |
| *moment of inertia - J* | 10000 | $kgm^2$ |
| *mass - $m_1$* | 100 | kg |
| *moment of inertia - $J_1$* | 10000 | $kgm^2$ |
| *mass - $m_2$* | 500 | kg |
| *moment of inertia - $J_2$* | 200000 | $kgm^2$ |
| *distance - a* | 1 | m |
| *distance - b* | 1 | m |
| *coefficient $\alpha$* | 2 | - |
| *stiffness - $k_1$* | 100...10000 | N/m |




(a)

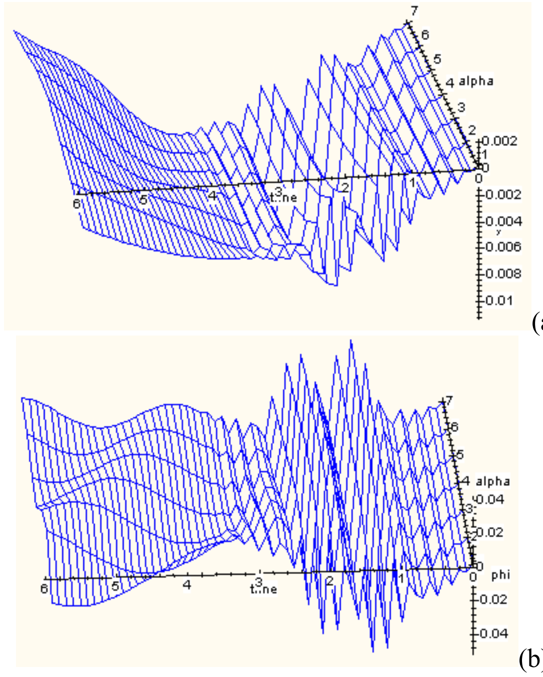
(b)

Figure 5. The influences of the α coefficient about the system displacements [a- y(t); b- φ(t)]

Based on the first proposed model (presented in the fig. 2) the author was analysis the influence of the coefficient α about the displacements y(t) and φ (t), and was observed that at the same time of the α growing, bring up the modulation magnitudes both for the linear, and for the angular movements.

Also, the eigen frequency for both deplacement types of the mass m acquire the high ranking values. These facts could be observed in the figure 5. In the first case was used the harmonical type of exciting force (eq. 15).

For the case of seismical wave form, the influences of the α coefficient about the deplacement shapes, could be view in the figure 6. Analysing this last set of diagrams, it could be say that the influence of the geometrical non-linearity are piffling in the time of external load acting, after that the system evolve to the stability in different ways, as the α coefficient has null or not.




(a)

(b)

Figure 6. The influences of the α coefficient about the system displacements, in the case of seismical type load [a- y(t); b- φ(t)]

Eigen values and eigen vectors for the presented system was computed with the well known expresion

$$\left(K - \lambda^2 M\right)\{\mu\} = \{0\} \tag{10}$$

and was obtained the next values

$$\lambda = [3{,}003363 \quad 0{,}026637] \tag{11}$$

$$\mu = \begin{bmatrix} 9{,}99435 & 0{,}33613 \\ -0{,}33613 & 0{,}999435 \end{bmatrix} \tag{12}$$

The influences of the real stiffness characteristic (eq. 13) was analysed through consider the k' value constant and impose that the k" stffness value have a variance between [0, 20000] N/m$^3$. In this case it was considered the second proposed model - presented in the figure 3.

The result of the numerical simulation for this non-linear analysis is presented in figure 7.

Natural values and eigen vectors for the presented system was computed with the expresion

$$\left(K - \lambda^2 M\right)\{\mu\} = \{0\} \tag{13}$$

(similar with the eqn. 10 of the first model) and was obtained the next values

$$\lambda = \left|3{,}6035 \quad 0{,}028 \quad -0.13\,10^{-9} \quad -0.12\,10^{-12}\right| \tag{14}$$

$$\mu = \begin{bmatrix} 9{,}98 & 0{,}029 & 0{,}848 & 0{,}0132 \\ -0{,}0033 & 0{,}0999 & 0{,}00055 & -0{,}0095 \\ 0{,}196 & -0{,}0059 & 0{,}848 & 0{,}0132 \\ -0{,}000165 & -0{,}005 & 0{,}00055 & -0{,}0095 \end{bmatrix} \tag{15}$$

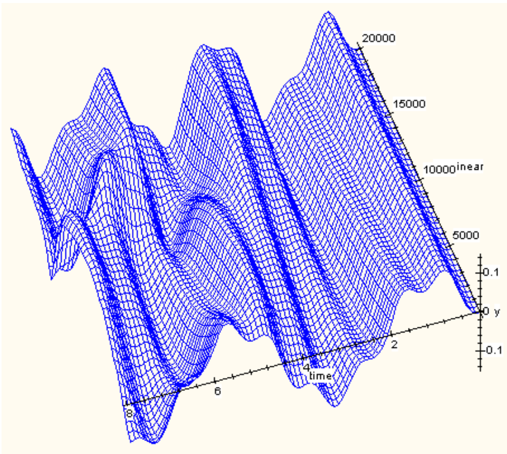
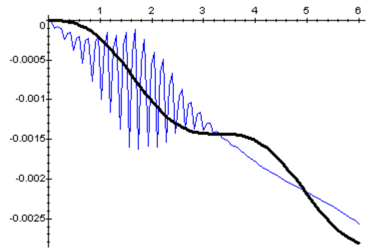
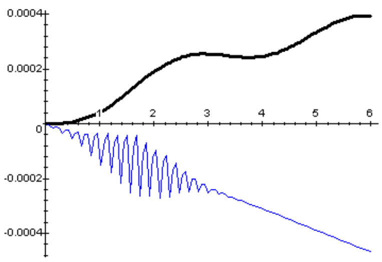



Figure 7. The influences of stiffness non-linearity on the system displacements



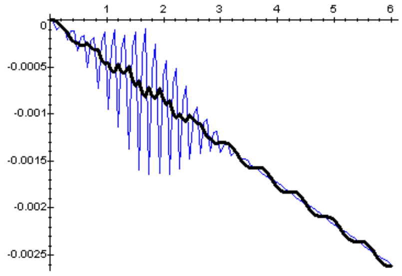
(a)



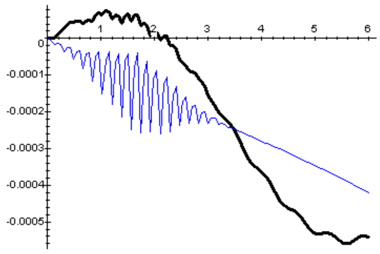
(b)

Figure 8. Temporal evolution of the system displacements for k$_1$=100N/m

In the figures 8, 9, 10 it is presented a set of diagrams for the vertical deplacement - *a* - and of the angular movement of the gravity center - *b*, with considering the case of the seismical wave form load (eq. 16) - simulations was developed on second model. The difference between the three cases consist from the k$_1$ value. It was presented the same type of deplacement in the same diagram, thus: thick line for $y_1(t)$, respective $\varphi_1(t)$, and sleazy line for $y_2(t)$, respective $\varphi_2(t)$.

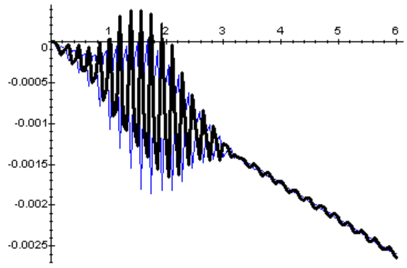


(a)

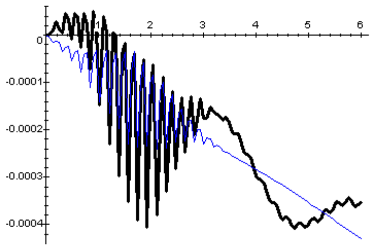


(b)

Figure 9. Temporal evolution of the system displacements for k1=3000N/m



(a)



(b)

Figure 10. Temporal evolution of the system displacements for k1=20000N/m

Certainly that the signal of mass m$_2$, where it was applied the external force, have approximate the same movement like seismical wave, but it could be viewed that the mass $m_1$ displacements are strongly influenced by the $k_1$ and $k_2$ stiffness values. At one time with increasing the stiffness values, growing the relative magnitude of the $y_2(t)$, respective $\varphi_2(t)$ signals (figure 10).

## 4. Concluding Remarks

Taking into account the numerical results, a sets presented in this paper, and considering that the model will be tunned with experimental data, we could say that this physical and mathematical model are very useful for analysing the dynamic behaviour of the passive elastic isolation system against the nocive effects of the vibrations or seismical waves. Also, this model must be completed with a spectral analysis for obtained exactly informations about the spectral composition of the $y(t)$ and $\varphi(t)$ signals. This informations help designers to avoid the dangerous phenomenon of resonance, that could be appear not only for the eigen value of system frequency, and for the high values of external loads frequencies that concur with the superior harmonics of the system.

This paper is a part of a large research study which consist by the analysis, both mathematical - numerical, and experimental, of antiseismic and antivibrational passive isolation systems, with multiple special configurations, taking into account the real characteristics of external loads and of rubber elements from the isolation system structure. The final purpose of this study consist by achieving all necessary information about this kind of passive isolation system, thus that, these could be used in any place where is imposed the need of protection against vibrations or seismic waves.

If these models will be tunned with existing experimental data, we could say that this physical and mathematical model are very useful for analysing the dynamic behaviour of the passive elastic isolation system against the nocive effects of the vibrations or seismical waves. And completed with a spectral analysis numerical routine, it getting very usefull for obtained exctly informations about the spectral composition of the $y(t)$ and $\varphi(t)$ signals. This informations help designers to avoid the dangerous phenomenon of resonance, that could be appear not only for the eigen value of system frequency, and for the high values of external loads frequencies that concur with the superior harmonics of the system.

## Acknowledgements

Numerical computations have been performed at the *Numerical Simulation of Processes Department*, and the behaviour analysis of the isolation systems have been performed at the *Dynamic Analysis of the Machines and Technical Equipments Department*, both from the *Engineering Faculty of Braila, National Research Center for Machines and Technological Equipments Mechanics*. Experimental analysis have been performed at the *Research Institute for Construction Equipment and Technologies* - ICECON SA Bucuresti.

## References

[1] **Bratu P.** - *Izolarea si amortizarea vibratiilor la utilaje de constructii*, Ed. INCERC, Bucuresti, 1982.

[2] **Bratu P., Bordea C.** - *Sistem elastic poligonal pentru izolarea socurilor si vibratiilor transmise de echipamentele dinamice inglobate in constructii*, Buletinul stiintific al celei de a XXVI-a Conferinte Nationale de Mecanica Solidelor, Braila, 14-15 iunie 2002.

[3] **Oproescu Gh., Nastac S.** - *Elemente de modelare numerica*, Ed. Libertatea, Braila, 2000.

[4] **Nastac S.** - *Analiza numerica cu aplicatii in ingineria mecanica*, Ed. IMPULS, Bucuresti, 2004.

[5] **Nastac S.** - *Analiza miscarii sistemului elastic poligonal de protectie antiseismica*, Buletinul Stiintific al celui de al IX-lea Simpozion National de Utilaje pentru Constructii, Bucuresti, 11-12 decembrie 2003.

[6] **Bratu P., Nastac S., Bordea C.** - *Analiza dinamica a sistemelor elastice antiseismice cu acordare joasa pentru izolarea bazei constructiilor*, Buletinul stiintific al celei de a XXVII-a Conferinte Nationale de Mecanica Solidelor, Pitesti, 23-24 mai 2003.

[7] **Bratu P., Nastac S., Bordea C.** - *Analiza dinamica a sistemelor elastice de protectie antiseismica pentru echipamentele vitale din dotarea constructiilor de interes public*, Buletinul stiintific al celei de a XXVII-a Conferinte Nationale de Mecanica Solidelor, Pitesti, 23-24 mai 2003.

[8] **Bratu P., Nastac S., Bordea C.** - *Sistem elastic antivibratil si antiseismic in configuratie poligonala*, Buletinul Stiintific al celui de al VIII-lea Simpozion National de Utilaje pentru Constructii, Bucuresti, 12-13 decembrie 2002.

[9] **Bratu P., Nastac S., Bordea C.** - *Anti-seismic Protection Solution for vital equipments from public buildings endowment*, The International Conference Constructions 2003, Cluj-Napoca,16 -17 mai 2003.

[10] **Nastac S., Bordea C.** - *Cercetari experimentale asupra sistemelor pasive de protectie antiseismica in configuratie poligonala*, A III-a Sesiune de comunicări stiintifice a catedrei de mecanică tehnica si mecanisme - SIMEC 2004, UTCB - Facultatea de Utilaj Tehnologic, Bucuresti, 26 martie 2004.

[11] **Nastac S., Bordea C.** - *Performantele funcţionale ale dispozitivelor si sistemelor pasive de protectie antiseismică*, A III-a Sesiune de comunicari stiintifice a catedrei de mecanica tehnica si mecanisme - SIMEC 2004, UTCB - Facultatea de Utilaj Tehnologic, Bucuresti, 26 martie 2004.

[12] **Bordea C., Nastac S.** - *Sistem elastic in configuratie poligonala folosit la protectia antiseismica a echipamentelor*, Monitorul AROTEM, anul 7 nr. 4 octombrie - decembrie 2003.

[13] **Nastac S.** - *Non-Linear Stiffness Influences About the Dynamics of the Anti-Vibrational Passive Isolation Systems*, Romanian Journal of Acoustics and Vibration, anul 1 nr. 1 octombrie - 2004.