THE ANNALS OF "DUNAREA DE JOS", UNIVERSITY OF GALATI
FASCICLE XIV MECHANICAL ENGINEERING, ISSN1224-5615
2006

THE DYNAMIC MODELS FOR CONSTITUTIVE OF MECHANICS SISTEMS TO EQUIPMENTS TEHNOLOGICAL PROPELLED

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ABSTRACT

The work tockle the mechanics transmission with denticulate under dynamic aspects, achieved simple models and easy to used in special case a complex transmissions. Followed the model realization who can be used in case at movement transmissions of equipments tehnological propelled , for the study kinematic excitation inducted of road.

1. Introduction

The equipments tehnological propelled from most areas: buildings, agriculture, forest, oilers, etc., have in elements mechanics components what used for the conduction of energy from motor (thermic or electric) at organ of thing most used are transmissions with denticulare wheels. Dynamic modelasion of these components are necessary for emphaseze a one a transitory phenomenons of the transfer process of power between sourse of energy and the organ of thing.

2. Dynamic model for transmission with two denticulate weels

So presupposed a gearing wher at considered a gearing rigidity, note with k and the factor of amortization from gearing with c, scale who determined the elastics forces and the amortization of gearing. In fig. nr.1 are reprezented the gearing of modulation.

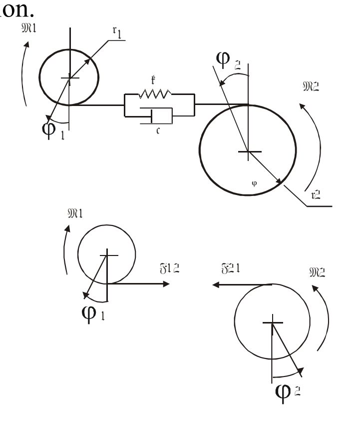


Fig. 1. The model of gearing of two denticulate wheels.

We apply the princypel D'Alembert for two denticulate wheels isolated and result the system of differential equation of motion.

$$\begin{cases} J_1 \dfrac{d^2\varphi_1}{dt^2} + cr_1(r_1 \dfrac{d\varphi_1}{dt} - r_2 \dfrac{d\varphi_2}{dt}) + \\ + kr_1(r_1\varphi_1 - r_2\varphi_2) = M_1 \\ J_2 \dfrac{d^2\varphi_2}{dt^2} - cr_2(r_1 \dfrac{d\varphi_1}{dt} - r_2 \dfrac{d\varphi_2}{dt}) - \\ - kr_2(r_1\varphi_1 - r_2\varphi_2) = M_2 \end{cases} \tag{1}$$

Below form of matrix system, she becomes:

$$[M].\{\ddot{q}\} + [C].\{\dot{q}\} + \{K\}.\{q\} = \{H\} \tag{2}$$

were:

$[M] = \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix}$ - the matrix of inertia;

$[C] = \begin{bmatrix} cr_1^2 & -cr_1r_2 \\ -cr_1r_2 & cr_2^2 \end{bmatrix}$ - the matrix of amortization;

$[K] = \begin{bmatrix} kr_1^2 & -kr_1r_2 \\ -kr_1r_2 & kr_2^2 \end{bmatrix}$ - the matrix of rigidity;

$\{q\} = \begin{Bmatrix} \varphi_1 \\ \varphi_2 \end{Bmatrix}$ – the vector of displaced;

$\{H\} = \begin{Bmatrix} M_1 \\ M_2 \end{Bmatrix}$ - vector of solicitations.

Consider k_m - the average rigidity of gearing; with who result the system as angular pulsation at gearing, that so:

$$\begin{aligned}&(k_m r_1^2 - p^2 J_1)(k_m r_2^2 - p^2 J_2) - \\ &- k_m^2 r_1^2 r_2^2 = 0;\end{aligned} \quad (3)$$

From relation (3) result the angular pulsation of free vibration on gearing, respective: p^2 =0 and $p^2 = k_m(r_1^2/J_1 + r_2^2/J_2)$, were result:

$$p_2 = r_1 . \sqrt{k_m (J_2 + i^2 J_1)/J_1 J_2}; \quad (4)$$

From analysis of requive at Stand regime of gearing, result:

$$k_m \begin{bmatrix} r_1^2 & -r_1 r_2 \\ -r_1 r_2 & r_2^2 \end{bmatrix} \begin{Bmatrix} \varphi_{10} \\ \varphi_{20} \end{Bmatrix} = \begin{Bmatrix} M_1 \\ M_2 \end{Bmatrix}; \quad (5)$$

were, k_m – represent the midlle rigidity of gearing, rigidity who is consider in to model as follows.

3. The dynamic model for transmission of system with many most denticulate wheels

Consider the model of mechanic transmission in n stairs of speed, represent in fig. 2.

The case considered deduced the system of motion wich departing from the simple model of transmyssion with a gearing generalizing it for the model with were gearing, specifics used an tehnological equipments.

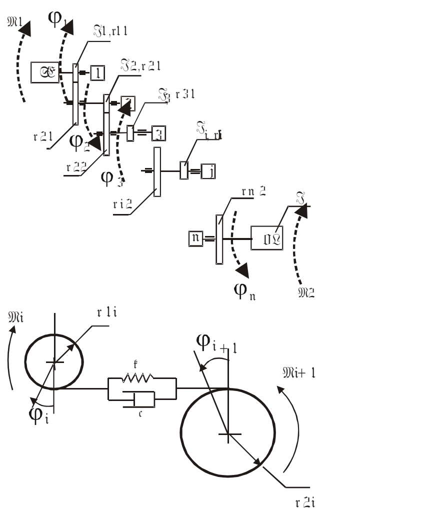


Fig. 2. The model of transmission with finit number of gearings

Note the transmission report of system like that:

$$i_1 = r_{21}/r_{11} = \varphi_1/\varphi_2; i_2 = r_{22}/r_{12} = \varphi_3/\varphi_2; ...$$

$$..i_i = r_{2i}/r_{1i} = \varphi_i/\varphi_{i-1}; ...i_n = r_{2n}/r_{2n-1} = \varphi_n/\varphi_{n-1};$$

Apply the principle of D'Alembert for the pair of denticulate wheels founded in gearing result the system of equation specific of number of gearing generalizing.

- The system in one stair. (i=1)

The transmision behave is like a mechanic system with two degrees of freedom and is mould as system of equation differential from down.

$$\begin{cases} J_1\ddot{\varphi}_1 + c.r_{11}.(r_{11}\dot{\varphi}_1 - r_{21}\dot{\varphi}_2) + \\ + k.r_{11}.(r_{11}\varphi_1 - r_{21}\varphi_2) = M_1 = M_s \\ J_2\ddot{\varphi}_2 - c.r_{21}.(r_{11}\dot{\varphi}_1 - r_{21}\dot{\varphi}_2) - \\ - k.r_{21}.(r_{11}\varphi_1 - r_{12}\varphi_2) = M_2 = -M_{OL} \end{cases}; \quad (6)$$

- The system in two stair. (i=2)

The transmision behave is like a mechanic system with three degrees of freedom and is mould as system of equation differential from down.

$$\begin{cases} J_1\ddot{\varphi}_1 + c.r_{11}.(r_{11}\dot{\varphi}_1 - r_{21}\dot{\varphi}_2) + \\ + k.r_{11}.(r_{11}\varphi_1 - r_{21}\varphi_2) = M_1 \\ J_2\ddot{\varphi}_2 - c.r_{21}.(r_{11}\dot{\varphi}_1 - r_{21}\dot{\varphi}_2) - \\ - k.r_{21}.(r_{11}\varphi_1 - r_{21}\varphi_2) + \\ + c.r_{12}.(r_{12}\dot{\varphi}_2 - r_{22}\dot{\varphi}_3) + \\ + k.r_{12}.(r_{12}\varphi_2 - r_{22}\varphi_3) = 0 \\ J_3\ddot{\varphi}_3 - c.r_{22}.(r_{12}\dot{\varphi}_2 - r_{22}\dot{\varphi}_3) - \\ - k.r_{22}.(r_{12}\varphi_2 - r_{22}\varphi_3) + M_{OL} = 0 \end{cases}; \quad (7)$$

- The system in three stair. (i=3)

The transmision require is like a mechanic system with four degrees of freedom and is mould as system of equation differential from down.

$$\begin{cases} J_1\ddot{\varphi}_1 + c.r_{11}.(r_{11}\dot{\varphi}_1 - r_{21}\dot{\varphi}_2) + \\ + k.r_{11}.(r_{11}\varphi_1 - r_{21}\varphi_2) = M_1 \\ J_2\ddot{\varphi}_2 - c.r_{21}.(r_{11}\dot{\varphi}_1 - r_{21}\dot{\varphi}_2) - \\ - k.r_{21}.(r_{11}\varphi_1 - r_{21}\varphi_2) + \\ + c.r_{12}.(r_{12}\dot{\varphi}_2 - r_{22}\dot{\varphi}_3) + \\ + k.r_{12}.(r_{12}\varphi_2 - r_{22}\varphi_3) = 0 \\ J_3\ddot{\varphi}_3 - c.r_{22}.(r_{12}\dot{\varphi}_2 - r_{22}\dot{\varphi}_3) - \\ - k.r_{22}.(r_{12}\varphi_2 - r_{22}\varphi_3) + \\ + c.r_{13}.(r_{13}\dot{\varphi}_3 - r_{23}\dot{\varphi}_4) + \\ + k.r_{13}.(r_{12}\varphi_3 - r_{23}\varphi_4) = 0 \\ J_4\varphi_4 - c.r_{23}.(r_{13}\dot{\varphi}_3 - r_{23}\dot{\varphi}_4) - \\ - k.r_{23}.(r_{13}\varphi_3 - r_{23}\varphi_4) + M_{OL} = 0 \end{cases};$$

In a similary way deduced the system of motion for many system in degrees if is necessary a such mechanism structure in dynamic study of the system a require of one tehnological equipment.

- The matrix form of a transmision model

The expresion of matrix for the general case of transmission with n gearing is:

$$[M]_i.\{\ddot{q}\} + [C]_i.\{\dot{q}\} + [K]_i.\{q\} = \{H\}_i\,; \quad i=1,2\ldots.n\,, \tag{8}$$

were the index i represent the number of gearing stairs. The semnification of terms is the same with other previous present, wich through generalization becames:

- the matrix of inertia have the expresion:

$$[M]_i = \begin{Vmatrix} J_1 & 0 & \ldots\ldots\ldots\ldots. & 0 \\ 0 & J_2 & 0 & 0 \\ \ldots. & \ldots. & \ldots\ldots & 0 \\ 0 & 0 & 0 & J_i \end{Vmatrix}; \tag{9}$$

- the matrix of amortization have the expresion:
-

$$[C]_i = cr_{11}^2 \begin{Vmatrix} 1 & -i_1 & .. & 0 & 0 \\ -i_1 & i_1^2 & .. & 0 & 0 \\ 0 & 0 & .. & 0 & 0 \\ .. & .. & .. & .. & .. \\ 0 & 0 & 0 & 0 & 0 \end{Vmatrix} + cr_{12}^2 \begin{Vmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -i_2 & .. & 0 \\ 0 & -i_2 & i_2^2 & .. & 0 \\ .. & .. & .. & .. & .. \\ 0 & 0 & 0 & 0 & 0 \end{Vmatrix} + . + cr_{1i-1}^2 \begin{Vmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ .. & .. & .. & .. & .. \\ 0 & 0 & 0 & 1 & -i_{i-1} \\ 0 & 0 & 0 & -i_{i-1} & i_{i-1}^2 \end{Vmatrix};$$

- the matrix of rigidity have the expresion:

$$[K]_i = kr_{11}^2 \begin{Vmatrix} 1 & -i_1 & .. & 0 & 0 \\ -i_1 & i_1^2 & .. & 0 & 0 \\ 0 & 0 & .. & 0 & 0 \\ .. & .. & .. & .. & .. \\ 0 & 0 & 0 & 0 & 0 \end{Vmatrix} + kr_{12}^2 \begin{Vmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -i_2 & .. & 0 \\ 0 & -i_2 & i_2^2 & .. & 0 \\ .. & .. & .. & .. & .. \\ 0 & 0 & 0 & 0 & 0 \end{Vmatrix} + . + kr_{1i-1}^2 \begin{Vmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ .. & .. & .. & .. & .. \\ 0 & 0 & 0 & 1 & -i_{i-1} \\ 0 & 0 & 0 & -i_{i-1} & i_{i-1}^2 \end{Vmatrix};$$

- the vector of acceleration, speed and require angular:

$$\{\ddot{q}\} = \begin{Vmatrix} \ddot{\varphi}_1 & \ddot{\varphi}_2 & .. & .. & \ddot{\varphi}_i \end{Vmatrix}^T; \quad \{\dot{q}\} = \begin{Vmatrix} \dot{\varphi}_1 & \dot{\varphi}_2 & .. & .. & \dot{\varphi}_i \end{Vmatrix}^T; \quad \{q\} = \begin{Vmatrix} \varphi_1 & \varphi_2 & .. & .. & \varphi_i \end{Vmatrix}^T; \tag{10}$$

- the vector of solicitations:

$\{H\}_i = \begin{Vmatrix} M_1 & 0 & 0 & .. & M_{OL} = M_i \end{Vmatrix}^T$; unde $\|..\|^T$ - who represent the matrix transpose of matrix with colums, form with coordinates generalizations proper.

- the pulsation equation for transmission with denticulate wheels is given by equation;

$$\det.\left|[K]_i - p^2[M]_i\right| = 0\,; \tag{11}$$

where: p- represent his pulsation of dynamic model analysed.

Achieved the proper calculation ti determine his pulsation for transmission with denticulate wheels, as folows:

- for transmission in one stairs - $p_{1,2}^2 = 0$;

$$p_{3,4}^2 = k.r_{11}^2.\left(\frac{1}{J_1} + \frac{i_1^2}{J_2}\right);$$

- for transmission in two stairs - $p_{1,2}^2 = 0$;

$$p_{3,4}^2 = k.r_{11}^2.\left(\frac{1}{J_1} + \frac{i_1^2}{J_2}\right); p_{5,6}^2 = k.r_{12}^2.\left(\frac{1}{J_2} + \frac{i_2^2}{J_3}\right);$$

- for transmission in three stairs -

$p_{1,2}^2 = 0$;

$$p_{3,4}^2 = k.r_{11}^2.(\frac{1}{J_1} + \frac{i_1^2}{J_2}); p_{5,6}^2 =$$

$$= k.r_{12}^2.(\frac{1}{J_2} + \frac{i_2^2}{J_3}); p_{7,8}^2 = k.r_{13}^2.(\frac{1}{J_3} + \frac{i_3^2}{J_4});$$

Is ascertain if pulsation of transmission an in principle the pulsation of stairs who composed the mechanic transmission with denticulate wheels. Relation of his pulsation can extend and for transmission with more stairs situation enough rarely meet in the case of transmission of tehnological equipments of construction autopropel.

4. The model for mechanic complex of transmission

The dinamic model present heve [1, 4s, took in consideration rigidity and amortization maked in gearing, imposed the mechanic models with many degreed of freedom what complicate very much the end of transmission model.

For avoid this draw back is necessary realisation a one dense dynamic model [4s for the components of mechanic transmission, model who carried reduction to the input axle of rigidity axles, of rigidity and amortization of gearing, neglected the momen of inertic of denticulate wheels.

This in the kind any constitutive mechanic of actuation, much as complicated I burn be, is boiled down to a system a mechanic with doua degrees of what freedom takes count of the rigidity and the equivalent amortization tuturor the components(arbors, dentitions). For illustrate is considered the system the mechanic from fig nr. 3 Format From engine, gearbox mechanic and the organ of thing.

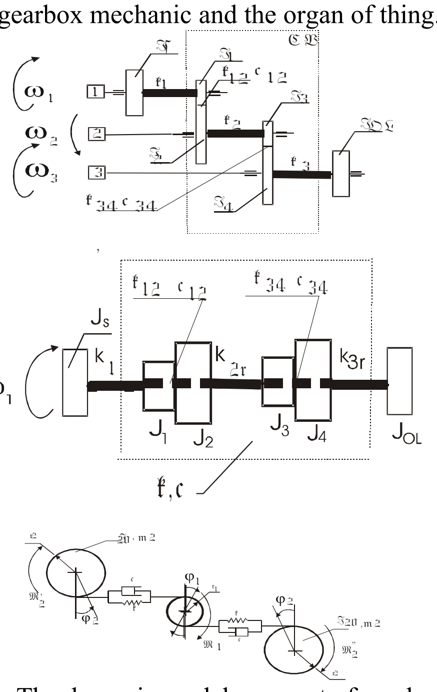


Fig. 3. The dynamic model compact of mechanic transmission

a) the real model; b) the compact model

For the propose model is determined the equivalent rigidity and the factors of amortization of structure a component of the box. From the calculus results:

$$\frac{1}{k} = \frac{1}{k_1} + \frac{i_1^2}{k_{12}} + \frac{i_1^2}{k_2} + \frac{i_1^2.i_2^2}{k_{34}} + \frac{i_1^2.i_2^2}{k_3};$$

$$\frac{1}{c} = \frac{i_1^2}{c_{12}} + \frac{i_1^2.i_2^2}{c_{34}}; \qquad (12)$$

From relation (12) is determine the equivalent rigidity of the box speed $k_{cv}= k$ and the amortization factor of the box $c_{cv}= c$. For exemplification is consider the case of mechanic transmission of the wheet back of some tehnological equipmnet transmission formed frou a speed of box (CV), transmission cardani (TC), differential (DF) and wheels.

In mould of mechanic transmission of whells back, motor, the take in considerent elasticitys and the factor of amortization of transmission components (gear box, cardan drive, axle the wheel). Becam the two wheels back she presupposed as workred simultaneously, the differential role. Is considered just below the appearance of elasticity and amortization entered from this in the system. Are not considerate the possibly difference of dynamic – kinectics entered from differential.

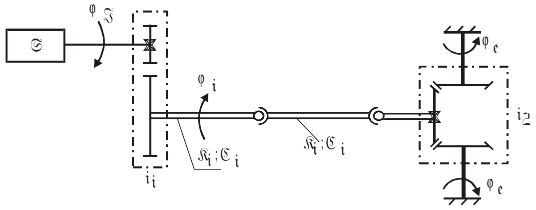


Fig. 4. The equivalent model of mechanic transmission

φ_I - the input angle in transmission; i_1 – the raport of conduction of the box; k_i – the elasticity of the transmission components; c_i – the amortization of transmission components; i_2 – the raport of differential conduction; φ_e - the angle of transmission of out; $\varphi_i - \varphi_I$ - the angle of rotation at transmission component; k_1 – the elasticity of the rank component; c_i – the amortization of component of rank; $\varphi_{ech_{TR}}$ - the angle of equivalent rotation; $k_{ech_{TR}} \cdot c_{ech_{TR}}$ - the characteristic equivalent of mechanic transmisiion

If consider that the system of mechanics transmission is formed from primary transmission component (I, i) and consider that the system is without exterior losses (conservativ), applying the energy theorem, result:

$$\frac{k_{ech} \cdot \varphi^2_{ech}}{2} = \sum_i \frac{k_i \varphi_i^2}{2}; \qquad (13)$$

and

$$\frac{c_{ech} \cdot \dot{\varphi}^2_{ech}}{2} = \sum_i \frac{c_i \dot{\varphi}_i^2}{2}; \qquad (14)$$

From (13) and (14) deduced characteristic equivalences of transmission changes, when results:

$$k_{ech} = \sum_i k_i \left(\frac{\varphi_i}{\varphi_{ech}}\right)^2; \text{ respectiv}$$

$$c_{ech} = \sum_i c_i \left(\frac{\dot{\varphi}_i}{\dot{\varphi}_{ech}}\right)^2; \qquad (15)$$

Because the report of transmission in to other mechanic transmission is defined through the relation:

$$i_i = \frac{\varphi_i}{\varphi_I} = \frac{\dot{\varphi}_i}{\dot{\varphi}_I}; \qquad (16)$$

he results as the relation (15) can write in the likeness of equivalence:

$$k_{ech} = \sum_i \frac{k_i}{i_i^2}; \text{ respectiv } c_{ech} = \sum_i \frac{c_i}{i_i^2}; \qquad (17)$$

where k_i and c_i are the respective rigidity amortization in the mechanic system of the gear box of the kinematic components (denticulate whels, arbors).

For the case considered the aplication face is deduced:

$$k_{ech_{CV}} = \frac{k_i}{i_i^2}; \text{ respectiv } c_{ech_{CV}} = \frac{c_i}{i_i^2}; \qquad (18)$$

where $k_{ech_{CV}}$ and $c_{ech_{CV}}$ represents the equident rigidity of the box of speed, respectively the factor of equivalent amortization of gear box.

For differentian is deduced the in the mode analogously:

$$k_{ech_{DP}} = \frac{k_i}{i_i^2}; \text{ respectively } c_{ech_{DF}} = \frac{c_i}{i_i^2}; \qquad (19)$$

where $k_{ech_{DF}}$ and $c_{ech_{DF}}$, represent the rigidity respectively the factor of equivalen amortization of diferential.

Becose the component of mechanic transmission are mounting in series (fig. no 4) hi result as for characteristic determination of rigidity and amortization of equivalence [2,7], for mountings series and parallel of elastic elements and the amortization.

$$\frac{1}{k_{ech_{TR}}} = \frac{1}{k_{ech_{CV}}} + \frac{1}{k_{ech_{TC}}} + \frac{1}{k_{ech_{DF}}};$$

$$\frac{1}{c_{ech_{TR}}} = \frac{1}{c_{ech_{CV}}} + \frac{1}{c_{ech_{TC}}} + \frac{1}{c_{ech_{DF}}}; \qquad (20)$$

From the relation (20) result rigidity and amortization equivalent of mechanic transmission considere, that is:

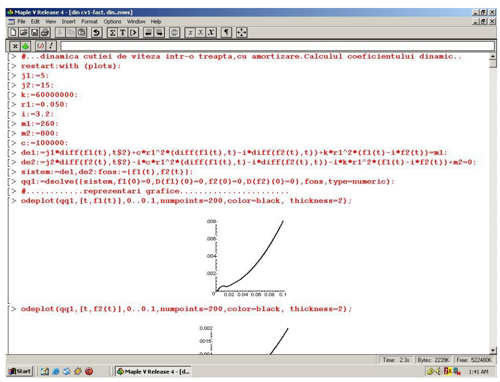
$$k_{TR} = k_{ech_{TR}} = \frac{k_{ech_{CV}} \cdot k_{ech_{TC}} \cdot k_{ech_{DF}}}{k_{ech_{TC}} \cdot k_{ech_{DF}} + k_{ech_{CV}} \cdot k_{ech_{DF}} + k_{ech_{CV}} \cdot k_{ech_{TC}}}; \qquad (21)$$

$$c_{TR} = c_{ech_{TR}} = \frac{c_{ech_{CV}} \cdot c_{ech_{TC}} \cdot c_{ech_{DF}}}{c_{ech_{TC}} \cdot c_{ech_{DF}} + c_{ech_{CV}} \cdot c_{ech_{DF}} + c_{ech_{CV}} \cdot c_{ech_{TC}}}; \qquad (22)$$

The model presented maybe be extensive to any transmission mechanic, with the condition evaluation of dynamic peculiar to each constitutive features of the system of the analysis.

5. Numerical model for transmissions mechanics

Analysed the dynamic of a behavior transmissions with denticulate wheels is considered an application particularizata for a transmission with a stair. The transmission has i= 3, 2, the rigidity and the factor of amortization have the determinate values experimentally [4s and represent the maxims values, what is decreased along with breed the number of stairs of gear. Modulation is achieved in Vmaple and the permited evaluation grafic form, depending on time with this sizes: the momentary angles of rotation (φ_1, φ_2), the angular speeds of rotation ($\dot{\varphi}_1, \dot{\varphi}_2$), the couple of inertia and the factor of dynamic behavior of the transmission (fig. no. 5, in order of the diagrams).



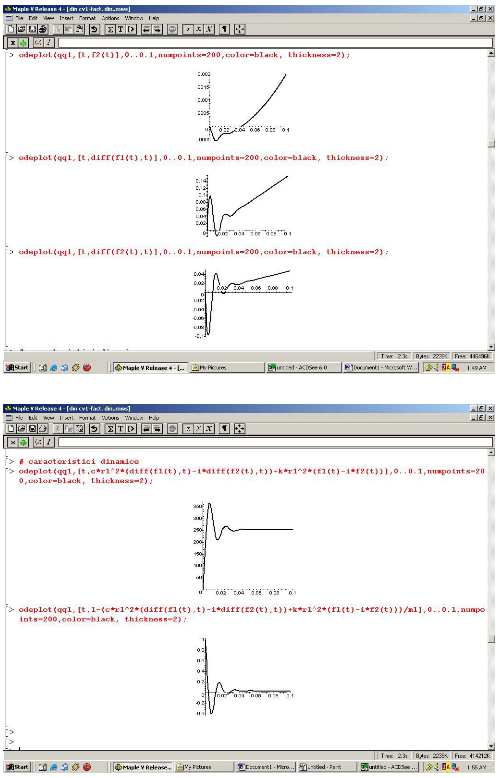


Fig. 5. The behavior of dynamic transmission

6. Conclusions

The analyse undertaked in the item have as the ultimate aim the finaly the elaboration of dybanic model for mechanic transmission complex which have account of the kinetic specific (angular speeds, report of conduction) and the characteristic of rigidity and the factor of amortization are entered from the axle and gearings formed the transmission.

In the trace of analysis result next appearances:

- the dyanmic model of transmission in one stairs is require like a mechanic system with two degrees of freedom whereat the rigidity and the factor of amortization are result from the compling in parallel of rigidity and amortizations entered be gearing and axle.
- the rigidity of gearing is value like midle rigidity of gearing (rel. 5).
- the complex transmission of system (box of speed), step down gear, differentials, etc) can be considerate all the mechanics system with two degrees of freedom gifts they have rigiditys and factors of equivalent amortization determinate through they coupling series or parallel of the groups of two gearing.
- the system of mechanic transmission with gearing is requive dybanic like the mechanic elasticity system with amortization, system who decrease this characteristics on measured what in transmission interfere more elements (more stairs of gearing or transmission components).
- the transmission mechanic of system are vulnerable of the phenomene of unsteadiness functional (rezonant), the when transmission are excited, toward exemple, from the dislevelments of the way of roll.

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