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LINEAR EQUIVALENT MECHANICAL OSCILLATORS WITH A NON-LINEAR OSCILLATOR AS A RESPONSE IN TIME

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ABSTRACT

The research of the non-linear mechanical oscillator on analytically way is not possible in the mostly cases because of the very difficult differential equations. The authors present a technique in order to find a linear equivalent oscillator as repose in the time and a concrete application.

1. Introduction

The movement of the non-linear oscillator is described from the differential equation [3]

$$m \cdot \ddot{x} + F_{fr}(x, \dot{x}) + F_{el}(x, \dot{x}) = F_{ex}(t) \qquad (1)$$

where m = mobile mass, x = displacement, F_{fr} = the dumping force, F_{el} = the elastically force, F_{ex} = the excited force.

The solution of the first equation, known also as the response in time of the non-linear oscillator, has two components: one transitory, that disappears in time and depends of the damping and one that is stationary that continues after the disappearance of the transitory.

We shall determine a linear mechanical oscillator that gives an equivalent response in time to the one given by the oscillator (1).

If the expressions of the forces (or their graphics) F_{fr}, F_{el} are non-linear, the response in the time of the oscillator after the stabilization of the movement, with

$$F_{ex} = F_0 \sin(\omega_0 t)$$

ω_0=the excitatory pulsation, F_0=the excitator amplitude, is periodic and non-harmonic as

$$x(t) = a_0 + \sum_{i=1}^{\infty} \left[a_i \cos(i\omega_0 t) + b_i \sin(i\omega_0 t)\right] \qquad (2)$$

The problem consist in the determination of a linear equivalent oscillator (as response in the time) and his equation has the form [2]

$$m_e \ddot{x} + c_e \dot{x} + k_e x = F(t) \qquad (3)$$

where m = equivalent mass, c = the equivalent dumping coefficient, k = the equivalent elastically coefficient, $F(t)$ =equivalent excitation.

2. Linear oscillator with equivalent response in time

2.1. Linear oscillator with extern excitation

If concerning a linear oscillator operate two or more harmonic excitation forces with different pulsation then the new movement will be the um of the movements that would have appeared if the forces would have been operated separately. That is correct because at the linear oscillators the superpose of the effects is valid.

The differential equation (3) is linear with constant coefficients and has the same solution (2) only if the excitation force is

$$F(t) = A_0 + \sum_{i=1}^{\infty}[A_i \cos(i\omega_0 t) + B_i \sin(i\omega_0 t)] \quad (4)$$

where A_0, A_i, B_i are quotients solved by identification for the named values of m_e, c_e, k_e.

From (2) it results

$$\dot{x}(t) = \varpi_0 \sum_{i=1}^{\infty} i[-a_i \sin(\varpi_0 t) + b_i \cos(i\varpi_0 t]$$

$$\ddot{x}(t) = -\varpi_0^2 \sum_{i=1}^{\infty} i^2[a_i \cos(\varpi_0 t) + b_i \sin(i\varpi_0 t]$$

and replacing in (3) we obtain:

$$\begin{aligned}&-m\omega^2 \cdot \sum_{i=1}^{\infty} i^2[a_i \cos(i\omega_0 t) + b_i \sin(i\omega_0 t)] + \\ &+ c\omega \cdot \sum_{i=1}^{\infty} i[-a_i \sin(i\omega_0 t) + b_i \cos(i\omega_0 t)] + ka_0 + \\ &+ k \cdot \sum_{i=1}^{\infty}[a_i \cos(i\omega_0 t) + b_i \sin(i\omega_0 t)] = \\ &= A_0 + \sum_{i=1}^{\infty}[A_i \cos(i\omega_0 t) + B_i \sin(i\omega_0 t)]\end{aligned}$$

Identifying

$$\begin{aligned}&A_0 = k \cdot a_0 \\ &A_i = \left(k - m\ \omega_0^2 i^2\right)a_i + (c\omega_0\, i)b_i \\ &B_i = -\left(c\ \omega_0\, i\right)a_i + \left(k - m\ \omega_0^2 i^2\right)b_i\ , \\ &i = 1,2,...\end{aligned} \quad (5)$$

where

$$a_0 = \frac{1}{T}\int_0^T x(t)dt$$

$$a_i = \frac{2}{T}\int_0^T x(t)\cos(i\omega_0 t)dt$$

$$b_i = \frac{2}{T}\int_0^T x(t)\sin(i\omega_0 t)dt$$

are the Fourier's coefficients [1], [5].

This way, the linear oscillator given by the third equation, where the excitation force is determined by (4) and (5), has the same stationary response in time, just like the non-linear one.

- if the two oscillators, the non-linear one and the linear equivalent one have the same mass m, for each couple of chosen values for the other coefficients c and k, it results a linear equivalent oscillator ; so to a non-linear oscillator we can associate ∞^2 different linear equivalent oscillators;
- if we consider linear equivalent oscillators with different mass, for any m, c, k we obtain a linear equivalent oscillator , so the equivalence of ∞^3 linear different oscillators with a non-linear one is possible.

This way to determine the linear equivalent oscillators can be also considered as a generalization for the linear techniques because it offers more solutions for solving the problem. Passing from a non-linear oscillator to a linear equivalent one is useful because the study of movements becomes easier and can be operated completely in a analytical way.

2.2 Linear non-excited oscillator (with free oscillation)

We can imagine many ways to create linear equivalent oscillators as a response in time for a non-linear oscillator. From the solution of the non-linear oscillator (2), operated in different ways and using the principle of the superposition of effects, we can determine more linear oscillators whom their summed movement can be identified with the non-linear oscillator movement. For the simplification of our work, these linear oscillators can be oscillators that have free movement, so without dumping and excitation, are described by the differential homogeneous equation as:

$$m_i \ddot{x} + k_i x = 0 \qquad (6)$$

has the solution $x_i(t)$ can lead to only one single solution :

$$x_{equiv}(t) = \sum_{i=1}^{n} x_i(t) \qquad (7)$$

which can be associated to the solution (2). This solution can be obtained if for each oscillator is assigned initial conditions (non-null values for the movement and / or initial speed) so that his solution can correspond to solution (2). It results the couple of values, m_i and k_i so that the pulsation of the *i*, type (6) oscillator can be equal with the pulsation found at the i pulsation component from (2)

$$\frac{k_i}{m_i} = (i\omega_0)^2 \;, \; i = 1,2,.... \qquad (8)$$

and the initial conditions, for example the initial displacement $x_{i\,0}$ and the initial speed $\dot{x}_{i\,0}$ can be obtained from (2).

$$x_{i\,0} = a_i \cos(i\omega_0 t_0) + b_i \sin(i\omega_0 t_0)$$

$$\dot{x}_{i\,0} = -a_i i\omega_0 \sin(i\omega_0 t_0) + b_i i\omega_0 \cos(i\omega_0 t_0)$$

3. Results

We have chosen a non-linear oscillator with m= 100kg, ω_0 = 100s^{-1} , F_0 = 10000N, with non-linear forces.

$$F_{fr} = \begin{cases} 5\cdot 10^3 \dot{x} + 2\cdot 10^6 \dot{x}^2 \;, & \dot{x} > 0 \\ 5\cdot 10^3 \dot{x} & , \; \dot{x} \le 0 \end{cases} ;$$

$$F_{el} = \left(10^6 |x| + 10^{11} x^2\right) \mathrm{sgn}(x)$$

where we can distinguish the asymmetry of the damping force. The solution of the equation (1), obtained by numerical integration is show in fig. 1. For different values of the number of components from solution (2), the stabilized responses obtained within the shown methods are presented in fig. 2, 3 and 4.

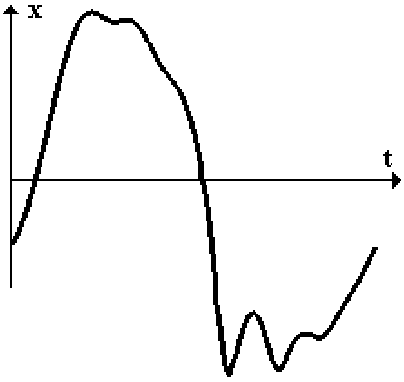

Fig. 1, Solutions of the equation (1)

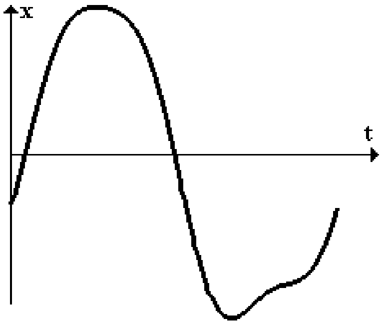

Fig. 2, Approximate solution for $i = 1 \cdots 3$

If the number of the terms of the correct solution increases, the approximation is better.

Fig. 3, Approximate solution for $i = 1 \cdots 5$

And because the solutions obtained within the two methods do not distinguish themselves , it is presented only a solution for each case.

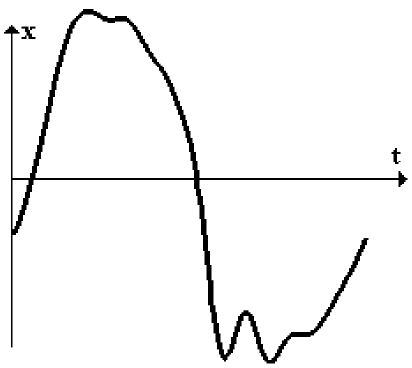

Fig. 4, Approximate solution for $i = 1 \cdots 10$

For the amplitude of 0,368 mm, the medium deviations are 0,04mm, 0,02809 mm, respectively 0,01986 mm.

4. Conclusions

We demonstrate that is possible to have a linear oscillator with the repose (as function dependent from time) equivalent to a non-linear oscillator. In order to obtain a linear equivalent oscillator we have used, as own techniques, the decomposition of the really reposes of the non-linear oscillator in Fourier's series.

The terms of the Fourier's series must verify the proposed linear equation and, from this condition, we discover the coefficients of the linear equation.

The accuracy of the equivalence depends from the number of the terms from the Fourier's series and can be made any precision. The results describe a really equivalent linear oscillator, their repose simulates with high precision the entire action of the non-linear oscillator. The constant terms from the linear equation as mass, friction and elastically coefficients can be chosen with any values. This fact permits a treble infinity variant of the linear oscillators.

In principle, a linear equivalent oscillator permits the analysis of the behaviour of the really non-linear oscillator on the analytically way in order to have a complete and generalised research of the non-linear phenomena. But each modification in the structure of the non-linear oscillator described from (1) has as effect another form for (2) and, finally, another linear equivalent oscillator.

This fact demonstrates a very important conclusion: a non-linear oscillator can be approximated with a linear oscillator, with any high precision, only punctually, for a concrete case. An approximation of the non-linear oscillator on a range of values of his parameters is not possible, the resulted linear oscillators are de facto another oscillator, with another characteristics, considered as equivalent on subjective arguments.

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