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THE SPATIAL VIBRATION OF MILLING MACHINE -SYNTHESIS-

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ABSTRACT

In this work a mathematical model is suggested with a view to the study of dynamic behavior of a horizontal milling machine, under the action of the three cutting force components, of which F_y produces the horizontal vibration of the machine, while F_x and F_z determine a vertical vibration. First case, the machine behaves like an elastic system with 3 degrees of freedom, while in the second case, the elastic system of the machine has 4 degrees of freedom. This analysis opens up with the dynamic concentration of the distributed mass. The deformed position of the machine might be obtained as a result of successive rotations. Starting from the system of fixed axes, it is successively obtained the systems of moving axes, solidly bound to various parts of the machine. The expressions of the reference axes unit vectors were determined by using the quaternion rotation operator. The orthogonal changes of coordinates are operating by using positional affine orthogonal tensors. The differential equations of the small vibrations of the system are analyzed by means of Lagrange's quadratic equations.

Notations

F_x, F_y, F_z -cutting force components [N]; θ, φ -angles [rad.]; $\omega, \dot{\theta}, \dot{\varphi}$ -angular speed [s^{-1}]; m, M - masses [kg]; a, b, c,..., l, m, n -lengths [m]; ρ -radius of gyration [m]; $J_{C\,x}$ -inertia moment to Cx axis [kg.m^2]; $\bar{i}, \bar{j}, \bar{k}$ -reference axis unit vectors; q -quaternion rotation operator; * -quaternion product; $T^{(i,j)}$ -affine orthogonal tensor; $x_n^{\tilde{k}}, y_n^{\tilde{k}}, z_n^{\tilde{k}}$ - k sphere center coordinates to x_n y_n z_n system of axes [m]; E -kinetic energy [N.m]; A, B -coefficients; D -energy dissipation function [N.m]; E_p -potential energy [N.m]; $F(t)$ -generalized force; K -elasticity constant [N.m]; C -damping factor [N.s.m^{-1}]; δL -virtual elementary mechanical work [N.m]; δ - differential quantity symbol.

1. Introduction

In this work a mathematical model is suggested with a view to the study of dynamic behavior of a horizontal milling machine (fig. 1), under the action of the three cutting force components (fig. 2), of which F_y produces the horizontal vibration of the machine (fig. 3), while F_x and F_z determine a vertical vibration (fig. 4).

Pictures 3 and 4 are deduced on the basis of experimental researches in [13].

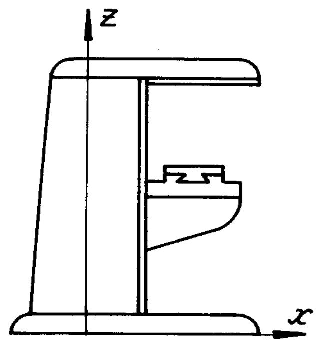

Fig. 1.

First case (fig. 3), the machine behaves like an elastic system with 3 degrees of freedom: θ_1, θ_2, θ_3, while in the second case(fig. 4), the elastic system of the machine

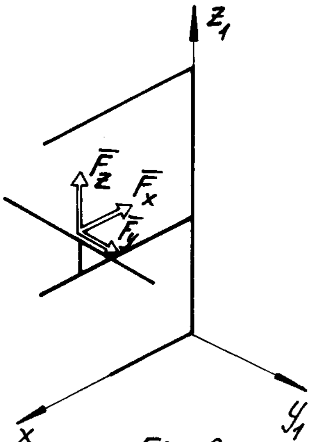

Fig. 2.

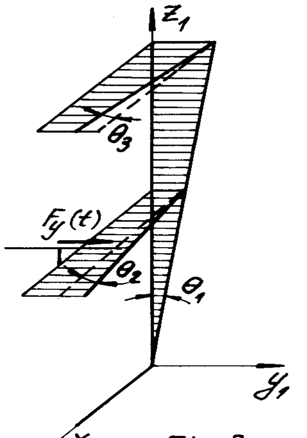

Fig. 3.

has 4 degrees of freedom: φ_1, φ_2, φ_3, φ_4.

2. The Study of the Dynamic Behavior of the Milling Machine

2.1. The dynamic concentration of the distributed mass

This analysis opens up with **the dynamic concentration of the distributed mass** in spheres of finite dimensions, as can be see in fig. 5, in which 1-2 represents the bedplate, 2 - 3 - 9 the column, 3 - 4 - 8 the console, 5 - 6 - 7 the machine table, 9 - 10 the over-arm.

Because the horizontal vibration is preponderant, the concentration of the mass will be made for the bodies in this movement.

For the bedplate 1-2 (fig. 6) the equations for the dynamic concentration of the mass are:

$$\begin{gathered} m_1 + m'_2 = M_{1,2} \\ m_1\, d_1 - m'_2\, d'_2 = 0 \\ m_1\,(\rho^2_1 + d^2_1) + m'_2\,(\rho'^2_2 + d'^2_2) = J_{C1\,z} \end{gathered} \tag{1}$$

In the system (1) it is known: the mass of the bedplate $M_{1,2}$, the position of the center of gravity C_1, the axial inertia moment $J_{C1\,z}$; it is chosen the radius of gyration ρ_1 and obtain the unknown values m_1, m'_2, ρ'_2 (for a sphere,

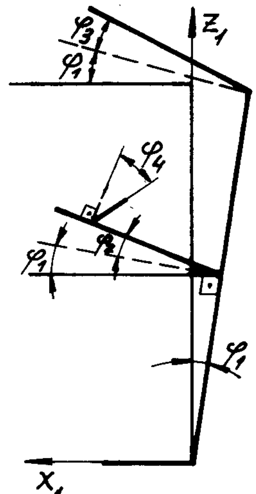

Fig. 4.

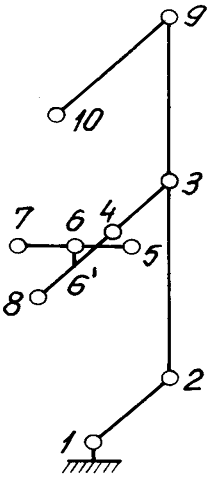

Fig. 5.

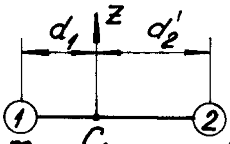

Fig. 6.

the radius of gyration is $\rho = R\,\sqrt{0,4}$).

For the column 2 - 3 - 9 (fig. 7), the equations are:

$$\begin{gathered} m''_2 + m'_3 + m'_9 = M_{2,9} \\ m'_3\, d'_3 + m'_9\, d'_9 - m''_2\, d''_2 = 0 \\ m''_2\,(\rho''^2_2 + d''^2_2) + m'_3\,(\rho'^2_3 + d'^2_3) + \\ + m'_9\,(\rho'^2_9 + d'^2_9) = J_{C2x}\,. \end{gathered} \tag{2}$$

We choose ρ''_2, ρ'_3, ρ'_9 and it is obtained m''_2, m'_3, m'_9.
Similar determinations are also made for the bodies 3 - 4 - 8, 5 - 6 - 7, 9 - 10. For the sphere 2 (fig. 5), there are:

$$m'_2 = m'_2 + m''_2\,;\quad \rho_2 = \rho'_2 = \rho''_2\,. \qquad (3)$$

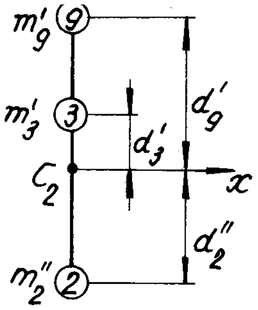

Fig.7.

An analogous calculus is made for the spheres 3 and 9.

2.2. The deformed position of the machine

The deformed position of the machine might be obtained as a result of successive rotations with the angles φ_1 and θ_1 (fig. 8), φ_2 and θ_2 (fig. 9), φ_3 and θ_3 (fig. 10), φ_4 (fig. 11).

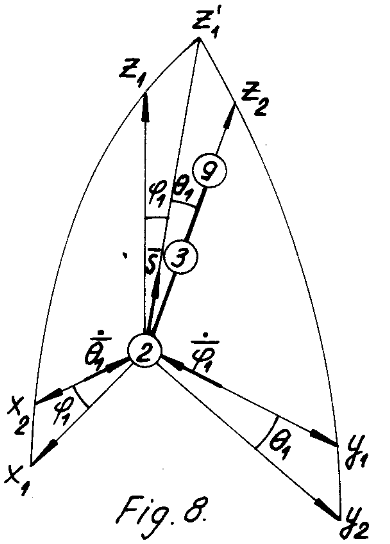

Fig. 8.

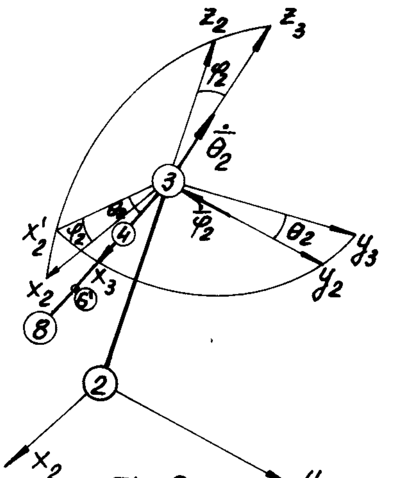

Fig. 9.

The rotations toke place at angular speeds $\dot{\varphi}_i$ *and* $\dot{\theta}_j$, $i = 1,2,3,4$; $j = 1, 2, 3$.

Starting from the system of fixed axes z_1 y_1 z_1 (fig. 2), it is successively obtained the systems of moving axes x_i y_i z_i, $i = 2, 3, 4$, solidly bound to various parts of the machine.

2.3. The expression of the reference axes unit vectors

The expression of the reference axes unit vectors are determined by using *the quaternion rotation operator* (see the Appendix A).

In this way, referring to figure 8, it is obtained:

$$\bar{i}_2 = q(\varphi_1,-\bar{j}_1)*\bar{i}_1 = (\cos\varphi_1 - \bar{j}_1\sin\varphi_1)*\bar{i}_1 =$$
$$= \bar{i}_1\cos\varphi_1 + \bar{k}_1\sin\varphi_1\,;$$
$$\bar{s} = q(\varphi_1,-\bar{j}_1)*\bar{k}_1 = (\cos\varphi_1 - \bar{j}_1\sin\varphi_1)*\bar{k}_1 =$$
$$= -\bar{i}_1\sin\varphi_1 + \bar{k}_1\cos\varphi_1\,;$$
$$\bar{k}_2 = q(\theta_1,-\bar{i}_2)*\bar{s} = (\cos\theta_1 - \bar{i}_2\sin\theta_1)*$$
$$*(-\bar{i}_1\sin\varphi_1 + \bar{k}_1\cos\varphi_1) = -\bar{i}_1\sin\varphi_1\cos\theta_1 +$$
$$+ \bar{j}_1\sin\theta_1 + \bar{k}_1\cos\varphi_1\cos\theta_1\,;$$
$$\bar{j}_2 = q(\theta_1,-\bar{i}_2)*\bar{j}_1 = (\cos\theta_1 - \bar{i}_2\sin\theta_1)*\bar{j}_1 =$$
$$= \bar{i}_1\sin\varphi_1\sin\theta_1 + \bar{j}_1\cos\theta_1 - \bar{k}_1\cos\varphi_1\sin\theta_1\,. \qquad (4)$$

For figure 9:

$$\bar{k}_3 = q(\varphi_2,-\bar{j}_2)*\bar{k}_2 = -\bar{i}_1\sin\varphi_2 + \bar{k}_2\cos\varphi_2\;;$$
$$\bar{i}'_2 = q(\varphi_2,-\bar{j}_2)*\bar{i}_2 = \bar{i}_1\cos\varphi_2 + \bar{k}_2\sin\varphi_2\;;$$
$$\bar{i}_3 = q(\theta_2,\bar{k}_3)*\bar{i}'_2 = \bar{i}_2\cos\varphi_2\cos\theta_2 + \bar{j}_2\sin\theta_2 +$$
$$+ \bar{k}_2\sin\varphi_2\cos\theta_2\;;$$
$$\bar{j}_3 = q(\theta_2,\bar{k}_3)*\bar{j}_2\,. \qquad (5)$$

We further similarly proceed to figures 10 and 11.

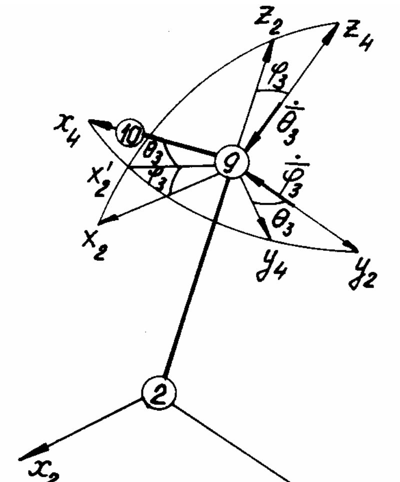

Fig. 10.

2.4. Positonal affin orthogonal tensors

The positonal affin orthogonal tensors which operate the orthogonal change of coordinates

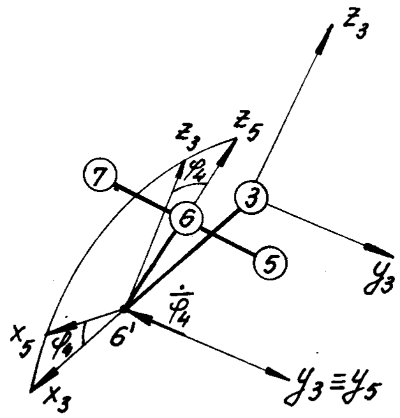

Fig. 11.

(see Appendix B), are:

$$T^{(1,2)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ x_1 & \bar{i}_2.\bar{i}_1 & \bar{j}_2.\bar{i}_1 & \bar{k}_2.\bar{i}_1 \\ y_1 & \bar{i}_2.\bar{j}_1 & \bar{j}_2.\bar{j}_1 & \bar{k}_2.\bar{j}_1 \\ z_1 & \bar{i}_2.\bar{k}_1 & \bar{j}_2.\bar{k}_1 & \bar{k}_2.\bar{k}_1 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -sin\varphi_1 & sin\varphi_1.sin\theta_1 & sin\varphi_1.cos\theta_1 \\ 0 & 0 & cos\theta_1 & sin\theta_1 \\ 0 & cos\varphi_1 & -cos\varphi_1.sin\theta_1 & cos\varphi_1.cos\theta_1 \end{bmatrix} \tag{6}$$

where x_1, y_1, z_1 are the coordinates of the axis system origin $x_2\,y_2\,z_2$ in relation to the system $x_1\,y_1\,z_1$.

Similarly:

$$T^{(2,3)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ x_2 & \bar{i}_3.\bar{i}_2 & \bar{j}_3.\bar{i}_2 & \bar{k}_3.\bar{i}_2 \\ y_2 & \bar{i}_3.\bar{j}_2 & \bar{j}_3.\bar{j}_2 & \bar{k}_3.\bar{j}_2 \\ z_2 & \bar{i}_3.\bar{k}_2 & \bar{j}_3.\bar{k}_2 & \bar{k}_3.\bar{k}_2 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\varphi_2.cos\theta_2 & -cos\varphi_2.sin\theta_2 & -sin\varphi_2 \\ 0 & sin\theta_2 & cos\theta_2 & 0 \\ \ell_{23} & sin\varphi_2.cos\varphi_2 & -sin\varphi_2.sin\theta_2 & cos\varphi_2 \end{bmatrix} \tag{7}$$

The same for $T^{(2,4)}$ and $T^{(3,5)}$.

The formulas for changing coordinates may be written in a matrix form:

$$\begin{bmatrix} 1 \\ x_1^{\tilde{3}} \\ y_1^{\tilde{3}} \\ z_1^{\tilde{3}} \end{bmatrix} = T^{(1,2)}. \begin{bmatrix} 1 \\ 0 \\ 0 \\ \ell_{23} \end{bmatrix}, \quad \begin{bmatrix} 1 \\ x_2^{\tilde{4}} \\ y_2^{\tilde{4}} \\ z_2^{\tilde{4}} \end{bmatrix} = T^{(1,2)}.T^{(2,3)}. \begin{bmatrix} 1 \\ \ell_{34} \\ 0 \\ 0 \end{bmatrix},$$

$$\begin{bmatrix} 1 \\ x_1^{\tilde{6}} \\ y_1^{\tilde{6}} \\ z_1^{\tilde{6}} \end{bmatrix} = T^{(1,2)}.T^{(2,3)}.T^{(3,5)}. \begin{bmatrix} 1 \\ 0 \\ 0 \\ \ell_{6'6} \end{bmatrix}, \tag{8}$$

where the " k "sphere center coordinates are marked $x_i^{\tilde{k}}, y_i^{\tilde{k}}, z_i^{\tilde{k}}$ on the reference system $x_i\,y_i\,z_i$. The k sphere center coordinates which do not appear in (8) are determined in a similar way.

If we approximate $\sin\psi \approx \psi$ and $\cos\psi \approx 1$, for $\psi = \varphi$ or θ, in the position tensors and neglect the double and triplex angular products , we obtain:

$$T^{(1,2)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\varphi_1 & 0 & -\varphi_1 \\ 0 & 0 & 1 & \theta_1 \\ 0 & 1 & -\theta_1 & 1 \end{bmatrix},$$

$$T^{(1,4)} = T^{(1,2)}.T^{(2,4)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\ell_{29}.\varphi_1 & -\varphi_1 & 0 & -\varphi_1 \\ \ell_{29}.\theta_1 & \theta_3 & 1 & \theta_1 \\ \ell_{29} & 1+\varphi_3 & \theta_3-\theta_1 & 1-\varphi_3 \end{bmatrix},$$

$$T^{(1,5)} = T^{(1,2)}.T^{(2,3)}.T^{(3,5)} = T^{(1,3)}.T^{(3,5)} =$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -(\ell_{23}+\ell_{36'})\varphi_1 & -\varphi_1 & \varphi_1 & -\varphi_1 \\ \ell_{23}\theta_1+\ell_{36'}\theta_2 & \theta_2 & 1 & \theta_1 \\ (\ell_{23}+\ell_{36'})+\ell_{36'}\varphi_2 & 1+\varphi_2+\varphi_4 & -\theta_1-\theta_2 & 1-\varphi_2-\varphi_4 \end{bmatrix} \tag{9}$$

$T^{(2,3)}$, $T^{(2,4)}$, $T^{(3,5)}$, $T^{(1,3)} = T^{(1,2)}.T^{(2,3)}$ are similarly obtained .

Now we may calculate the sphere centers coordinates in relation to the fixed system of axes $x_1\, y_1\, z_1$: $x_1^{\ddot{k}}, y_1^{\tilde{k}}, z_1^{\tilde{k}}$, for k = 3, 4, 5,..., 10, as well as the speed of the sphere centers:

$$v_k^2 = (\dot{x}_1^{\tilde{k}})^2 + (\dot{y}_1^{\tilde{k}})^2 + (\dot{z}_1^{\tilde{k}})^2 . \qquad (10)$$

For example, it is obtained for the *6* sphere center the following:

$$\begin{bmatrix} 1 \\ x_1^{\tilde{6}} \\ y_1^{\tilde{6}} \\ z_1^{\tilde{6}} \end{bmatrix} = T^{(1,5)} \begin{bmatrix} 1 \\ 0 \\ 0 \\ \ell_{6'6} \end{bmatrix} , \qquad (11)$$

where from:

$$\begin{aligned} \dot{x}_1^{\tilde{6}} &= -(\ell_{23} + \ell_{36'} + \ell_{6'6})\dot{\varphi}_1 \\ \dot{y}_1^{\tilde{6}} &= (\ell_{23} + \ell_{6'6})\dot{\theta} + \ell_{36'}\dot{\theta}_2 \\ z_1^{\tilde{6}} &= (\ell_{36'} - \ell_{6'6})\dot{\varphi}_2 - \ell_{6'6}\dot{\varphi}_4 \end{aligned} \qquad (12)$$

2.5. The calculation of the spheres kinetic energies in their movement in relation to its very center

For sphere 3, for example, which participates in the rotations $\dot{\bar{\varphi}}_1, \dot{\bar{\theta}}_1, \dot{\bar{\varphi}}_2, \dot{\bar{\theta}}_2$ (fig. 12), it may be written, taking into account (4) and (5):

$$\begin{aligned} &\dot{\bar{\varphi}}_1 = -\dot{\varphi}_1 \bar{j}_1; \\ &\dot{\bar{\varphi}}_2 = -\dot{\varphi}_2 \bar{j}_2 = -\dot{\varphi}_2(\bar{i}_1 sin\varphi_1 sin\theta_1 + \\ &+ \bar{j}_1 cos\theta_1 - \bar{k}_1 cos\varphi_1 sin\theta_1; \\ &\dot{\bar{\theta}}_1 = -\dot{\theta}_1 \bar{i}_2 = -\dot{\theta}_1(-\bar{i}_1 sin\varphi_1 + \bar{k}_1 cos\varphi_1); \\ &\dot{\bar{\theta}}_2 = \dot{\theta}_2 \bar{k}_3 . \end{aligned} \qquad (13)$$

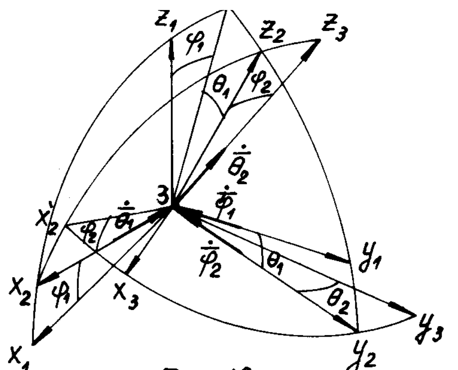

Fig. 12.

The resultant angular speed of the sphere *3* is:

$$\bar{\omega}_3 = \dot{\bar{\varphi}}_1 + \dot{\bar{\varphi}}_2 + \dot{\bar{\theta}}_1 + \dot{\bar{\theta}}_2 , \qquad (14)$$

with projections on the axes of fixed system:

$$\begin{aligned} \omega_{3x} &= -\dot{\varphi}_2 \sin\varphi_1 \sin\theta_1 + \dot{\theta}_1 \sin\varphi_1 - \\ &- \dot{\theta}_2 (\sin\varphi_1 \cos\varphi_2 + \cos\theta_1 \sin\varphi_1 \sin\varphi_2); \\ \omega_{3y} &= -\dot{\varphi}_1 - \dot{\varphi}_2 \cos\theta_1 + \dot{\theta}_2 \sin\theta_1 \sin\varphi_2 ; \\ \omega_{3z} &= \dot{\varphi}_2 \cos\varphi_1 \sin\theta_1 - \dot{\theta}_1 \cos\varphi_1 - \\ &- \dot{\theta}_2 (\cos\varphi_1 \cos\varphi_2 + \cos\theta_1 \cos\varphi_1 \sin\varphi_2). \end{aligned} \qquad (15)$$

We also operate in (15) the approximation regarding the small angles, made in the paragraph 2. 4; moreover we neglect the products of type: $\dot{\psi}\psi$. The kinetic energy of sphere *3* obtained from its movement in relation to its center C_3 may be written:

$$\begin{aligned} E_{C_3} &= \frac{1}{2} J_3 (\omega_{3x}^2 + \omega_{3y}^2 + \omega_{3z}^2) = \\ &= \frac{1}{2} J_3 (\dot{\varphi}_1^2 + \dot{\varphi}_2^2 + 2\dot{\varphi}_1\dot{\varphi}_2 + \dot{\theta}_1^2 + \dot{\theta}_2^2 - 2\dot{\theta}_1\dot{\theta}_2) \end{aligned} \qquad (16)$$

2.6. The kinetic energy

The kinetic energy of the 10 spheres system:

$$E = \sum_{i=1}^{10} E_i = \sum_{i=1}^{10} (E_{C_i} + \frac{m_i v_i^2}{2}) , \qquad (17)$$

may be written as:

$$E = \sum_{i=1}^{7} A_i \dot{\psi}_i^2 + \sum_{i=1}^{7} \sum_{\substack{j=1 \\ i \neq j}}^{7} B_{ij} \psi_i \psi_j , \qquad (18)$$

considering: $\psi_i = \varphi_i$, for $i = 1, 2, 3, 4$ and $\psi_{i+4} = \theta_i$, for $i = 1, 2, 3$.

2.7. The differential equations

The differential equations of the small vibrations of the system are [6]:

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{\psi}_i}\right) + \frac{\partial D}{\partial \dot{\psi}_i} - \frac{\partial E}{\partial \psi_i} + \frac{\partial E_P}{\partial \psi_i} = F_i(t) ,$$
$$\text{for } i = 1, 2, \ldots, 7 , \qquad (19)$$

where E_P represent the potential energy of the system, while *D* is the energy dissipation function.

For the horizontal vibration of the machine (fig. 3), the column, the console and the over-arm may be considered rigid elements which are articulated in this way: the first to the bedplate, while the other two to the column, as in fig. 13, 14 and 15.

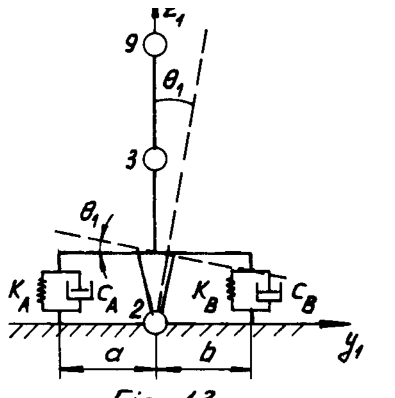

Fig. 13.

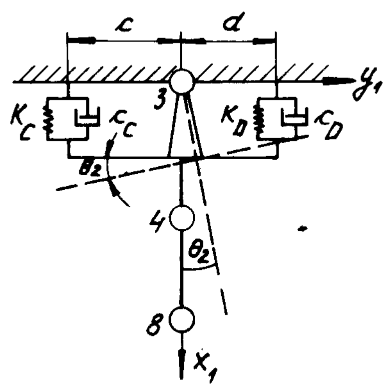

Fig. 14.

The potential energy of the system in horizontal vibration is:

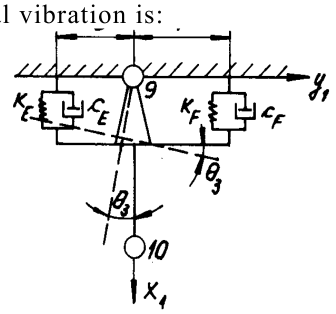

Fig. 15.

$$E'_P = \frac{K_A a^2 + K_B b^2}{2}\theta_1^2 + \frac{K_C c^2 + K_D d^2}{2}\theta_2^2 + \frac{K_E e^2 + K_F f^2}{2}\theta_3^2 \quad ,$$

or $$E'_P = \frac{1}{2}K_1\theta_1^2 + \frac{1}{2}K_2\theta_2^2 + \frac{1}{2}K_3\theta_3^2 \tag{20}$$

The energy dissipation function:

$$D' = \frac{C_A a^2 + C_B b^2}{2}\dot{\theta}_1^2 + \frac{C_C c^2 + C_D d^2}{2}\dot{\theta}_2^2 + \frac{C_E e^2 + C_F f^2}{2}\dot{\theta}_3^2 \quad , \tag{21}$$

or $$D' = \frac{1}{2}C_1\dot{\theta}_1^2 + \frac{1}{2}C_2\dot{\theta}_2^2 + \frac{1}{2}C_3\dot{\theta}_3^2$$

For the vertical vibration (fig. 4), the shifted position of the column, the console, the over-arm and the machine table are drawn in a dotted line in fig.16, 17, 18 and 19.

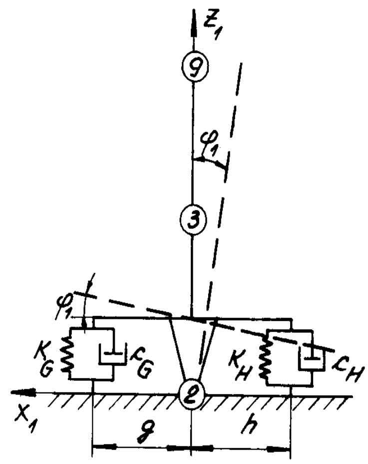

Fig. 16.

The potential energy of the system in this vibration is:

$$E''_P = \sum_{i=4}^{7}\frac{1}{2}K_i\,\psi_{i-3}^2 \tag{22}$$

where: $K_4 = K_G \,.\, g^2 + K_H \,.\, h^2$;
$K_5 = K_I \,.\, i^2 + K_J \,.\, j^2$, etc.

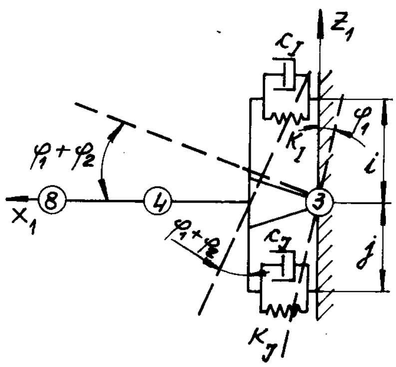

Fig. 17.

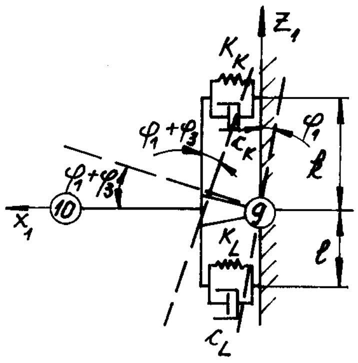

Fig. 18.

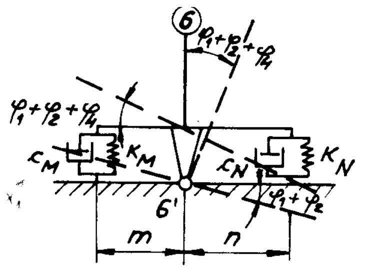

Fig. 19.

The energy dissipation function:

$$D'' = \sum_{i=4}^{7} \frac{1}{2} C_i \dot{\psi}_{i-3}^{2} \ ,$$

where $C_4 = C_G \, . \, g^2 + C_H \, . \, h^2$, etc. (23)

For the system in general vibration, the potential energy and the energy dissipation function are:

$$\begin{aligned} E_P &= E'_P + E''_P = \sum_{i=1}^{7} \frac{1}{2} K_i \psi_i^2 \quad ; \\ D &= D' + D'' = \sum_{i=1}^{7} \frac{1}{2} C_i \psi_i^2 \end{aligned} \quad , \qquad (24)$$

where is used the angles notations of (18).

We calculate the expressions in the equations (19):

$$\frac{\partial E}{\partial \dot{\psi}_i} = A_i \dot{\psi}_i + \sum_{\substack{j=1 \\ j \neq i}}^{7} B_{ij} \dot{\psi}_j = \sum_{j=1}^{7} M_{ij} \dot{\psi}_j \quad ;$$

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{\psi}_i} \right) = \sum_{j=1}^{7} M_{ij} \ddot{\psi}_j \quad ; \quad \frac{\partial D}{\partial \dot{\psi}_i} = C_i \dot{\psi}_i \quad ;$$

$$\frac{\partial E}{\partial \psi_i} = 0 \quad ; \quad \frac{\partial E_P}{\partial \psi_i} = K_i \psi_i$$

(25)

The generalized forces $F_i(t)$ from (19) may be determined by calculating the virtual elementary mechanical work of the forces $F_x(t)$, $F_y(t)$, $F_z(t)$ (fig.20), applied in point *6* (fig.6 and 20):

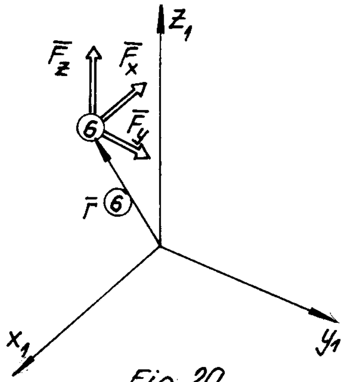

Fig. 20.

$$\begin{aligned} \delta L &= \overline{F}_x \delta \tilde{\bar{r}}^6 + \overline{F}_y \delta \tilde{\bar{r}}^6 + \overline{F}_z \delta \tilde{\bar{r}}^6 = \\ &= -F_x \delta \tilde{x}_1^6 + F_y \delta \tilde{y}_1^6 + F_z \delta \tilde{z}_1^6 \end{aligned} \quad , \qquad (26)$$

where we shall use the relations (11):

$$\begin{aligned} \delta \tilde{x}_1^6 &= -(\ell_{23} + \ell_{36'} + \ell_{6'6}) \delta \varphi_1 = -L_1 \delta \psi_1 \quad ; \\ \delta \tilde{y}_1^6 &= (\ell_{23} + \ell_{6'6}) \delta \theta_1 + \ell_{36'} \delta \theta_2 = \\ &= L_5 \delta \psi_5 + L_6 \delta \psi_6 \quad ; \\ \delta \tilde{z}_1^6 &= (\ell_{36} - \ell_{6'6}) \delta \varphi_2 - \ell_{6'6} \delta \varphi_4 = \\ &= L_2 \delta \psi_2 - L_4 \delta \psi_4 \quad ; \end{aligned}$$

(27)

so that:

$$\begin{aligned} \delta L &= F_x L_1 \delta \psi_1 + F_y L_5 \delta \psi_5 + F_y L_6 \delta \psi_6 + \\ &+ F_z L_2 \delta \psi_2 - F_z L_4 \delta \psi_4 \ , \end{aligned} \qquad (28)$$

where from: $F_1(t) = F_x L_1$; $F_2(t) = F_z L_2$; $F_3(t) = 0$; $F_4(t) = -F_z L_4$; $F_5(t) = F_y L_5$; $F_6(t) = F_y L_6$; $F_7(t) = 0$. (29)

The equations (19) may be written:

$$\sum_{j=1}^{7} M_{ij} \ddot{\psi}_j + C_i \dot{\psi}_i + K_i \psi_i = F_i(t) \ ,$$

$$i = 1, 2, \ldots, 7 \ . \qquad (30)$$

The system of differential equations (30) is turned into a system of algebraic equations if the Laplace Transformation is applied to it (considering the initial conditions as null), and the algebraic system obtained in this way may be written in a matrix form:

$$\begin{bmatrix} M_{11}s^2+C_1s+K_1 & . & . & M_{17}s^2 \\ M_{21}s^2 & . & . & M_{27}s^2 \\ M_{31}s^2 & . & . & M_{37}s^2 \\ . & . & . & . \\ . & . & . & . \\ M_{71}s^2 & . & . & M_{77}s^2+C_7s+K_7 \end{bmatrix} \cdot \begin{bmatrix} \psi_1(s) \\ \psi_2(s) \\ \psi_3(s) \\ . \\ . \\ \psi_7(s) \end{bmatrix} = \begin{bmatrix} F_1(s) \\ F_2(s) \\ F_3(s) \\ . \\ . \\ F_7(s) \end{bmatrix}$$

or in a symbolic notation :

$$\boldsymbol{W} \cdot \boldsymbol{\Psi} = \boldsymbol{F} \tag{31}$$

3. Conclusions

This matrix equation describes the automatic system of the general vibration for the milling machine, having the input vector $\boldsymbol{F}$ and the output vector $\boldsymbol{\Psi}$. From previous equation we obtain:

$$\boldsymbol{\Psi} = \boldsymbol{W}^{-1} \cdot \boldsymbol{F} = \boldsymbol{Y} \cdot \boldsymbol{F}$$

The matrix $\boldsymbol{Y}=\boldsymbol{W}^{-1}$ is called the transfer matrix of the system, used in the analysis of the dynamic behaviour of the machine, as well as in identification.

We notice that matrix $\boldsymbol{Y}$ describes the **open** automatic system of the machine. In order to analyze the **closed** (*real*) automatic system we must also determine the transfer matrix of the cutting process, a problem which the author intends to deal with in the future.

With this work we hope we answered to the request from{4}, pp. 626, which wanted us to describe the chatter of the milling machine like a spatial vibration.

APPENDIX A

From [14] we sum up the following:
If the vector $\bar{a}$ (fig. 21) is obtained by rotating the vector $\bar{b}$ with the angle θ, along the direction given by vector $\bar{v}$ (according to the right handed screw rule), then there is the relation: $\bar{a} = (\cos\theta + \bar{v}\sin\theta) * \bar{b}$
where:

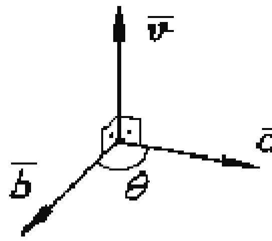

Fig. 21

- the vectors $\bar{b}$ and $\bar{v}$ are known by projecting them on a system of axes:

$$\bar{b} = b_1\,\bar{i} + b_2\,\bar{j} + b_3\,\bar{k}\ ;\ \bar{v} = v_1\,\bar{i} + v_2\,\bar{j} + v_3\,\bar{k}\ ;$$

- the sign $*$ symbolises the *quaternion product*, which is non commutative.

The quaternion product of the unit vectors is obtained according to the rule:

$$\bar{i}*\bar{i} = \bar{j}*\bar{j} = \bar{k}*\bar{k} = -1$$

$$\bar{i}*\bar{j} = \bar{k} \qquad \bar{j}*\bar{i} = -\bar{k}$$

$$\bar{j}*\bar{k} = \bar{i} \qquad \bar{k}*\bar{j} = -\bar{i}$$

$$\bar{k}*\bar{i} = \bar{j} \qquad \bar{i}*k = -\bar{j}$$

It is marked *the quaternion rotation operator*: $q(\theta,\bar{v}) = \cos\theta + \bar{v}\sin\theta$.

Theorem: $q(\theta,\bar{v})$ operates the rotation of a vector $\bar{b}$ round $\bar{v}$, with the angle θ, having as a result the vector $\bar{a}$, if it is applied on the left of the vector $\bar{b}$, in a quaternion product: $\bar{a} = q(\theta,\bar{v}) * \bar{b}$

APPENDIX B

From [8, 9, 12] we sum up the following:
Let be I_{v-1} and I_v two elements that belong to a linkage, between them being a cinematic coupling (fig. 22). It is knotted:

$$\alpha_{ij}^{(v,v-1)} = \cos(\bar{i}_i^{\,v}; \bar{i}_j^{\,v-1}) = \bar{i}_i^{\,v} \cdot \bar{i}_j^{\,v-1}$$

Between the coordinates of a point $M \in I_v$ and the coordinates of the same point belonging to I_{v-1}, we have got (fig. 23):

$$x_{v-1,j} = x_{v,j} + \alpha_{1j}^{(v,v-1)} x_{v1} + \alpha_{2j}^{(v,v-1)} x_{v2} + \alpha_{3j}^{(v,v-1)} x_{v3}\ ,\ (j = 1, 2, 3)\ . \tag{32}$$

These relations establish a biunique correspondence between the I_{v-1} and I_v domain points.

The system (32) can be replaced by a linear and homogeneous equations system, putting:

$$x_{v-1,j} = X_{v-1,j+1} / X_{v-1,1}$$
$$\text{and} \quad x_{v-1,j} = X_{v,j+1} / X_{v,1} \quad , (j = 1, 2, 3); \quad (33)$$

with $X_{v-1,1} = X_{v,1}$, (32) becomes : $X_{v-1,1} = X_{v,1}$,

$$X_{v-1,j+1} = x_j{}^{v} X_{v1} + \alpha_{1j}{}^{(v,v-1)} X_{v2} + \alpha_{2j}{}^{(v,v-1)} X_{v3} + \alpha_{3j}{}^{(v,v-1)} X_{v4}, \; (j = 1,2,3). \quad (34)$$

(34) define an **affin orthogonal change of coordinates**.

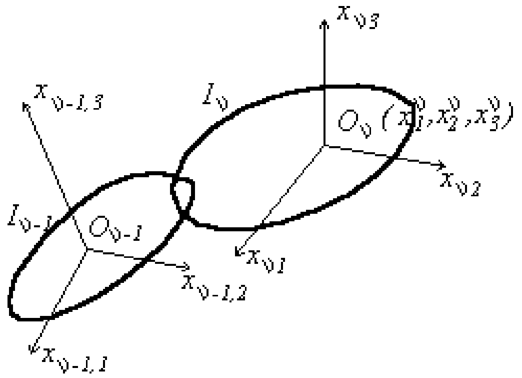

Fig. 22

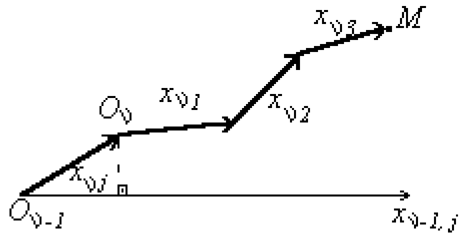

Fig. 23

Being in Euclid's space in four dimensions, introduced, above an **affin orthogonal tensor of order 2**, specified to the considered cinematic coupling, which realizes the orthogonal change given by (34):

$$T^{(\nu,\nu-1)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ x_1^\nu & \alpha_{11}^{(\nu,\nu-1)} & \alpha_{21}^{(\nu,\nu-1)} & \alpha_{31}^{(\nu,\nu-1)} \\ x_2^\nu & \alpha_{12}^{(\nu,\nu-1)} & \alpha_{22}^{(\nu,\nu-1)} & \alpha_{32}^{(\nu,\nu-1)} \\ x_3^\nu & \alpha_{13}^{(\nu,\nu-1)} & \alpha_{23}^{(\nu,\nu-1)} & \alpha_{33}^{(\nu,\nu-1)} \end{bmatrix} \quad (35)$$

The formulas of coordinates change can be transcribed as abbreviate matrix form:

$$\begin{bmatrix} 1 \\ \bar{r}_\nu \end{bmatrix} = T^{(\nu,\nu-1)} \begin{bmatrix} 1 \\ \bar{r}_{\nu-1} \end{bmatrix}, \quad \text{or} \begin{bmatrix} 1 \\ \bar{r}_{\nu-1} \end{bmatrix} = T^{(\nu-1,\nu)} \begin{bmatrix} 1 \\ \bar{r}_\nu \end{bmatrix} \quad (36)$$

in which $\bar{r}_\nu$ *and* $\bar{r}_{\nu-1}$ were knotted the position uni-column matrices of one and the same point *M* in the considered Cartesian coordinates systems. The schema of any mechanism can be considered as being formed of a linkage. The elements' positions analysis is made by associating to each element a Cartesian frame (fig. 22). By applying the equations (36) relative to the coordinates change of the systems bounded by successive elements, it is obtained:

$$\begin{bmatrix} 1 \\ \bar{r}_0 \end{bmatrix} = T^{(0,1)} \begin{bmatrix} 1 \\ \bar{r}_1 \end{bmatrix};$$

$$\begin{bmatrix} 1 \\ \bar{r}_1 \end{bmatrix} = T^{(1,2)} \begin{bmatrix} 1 \\ \bar{r}_2 \end{bmatrix}; \ldots; \begin{bmatrix} 1 \\ \bar{r}_{\lambda-1} \end{bmatrix} = T^{(\lambda-1,\lambda)} \begin{bmatrix} 1 \\ \bar{r}_\lambda \end{bmatrix}$$

from where , by successive substitutes :

$$\begin{bmatrix} 1 \\ \bar{r}_0 \end{bmatrix} = T^{(0,1)} T^{(1,2)} \ldots T^{(\lambda-1,\lambda)} \begin{bmatrix} 1 \\ \bar{r}_\lambda \end{bmatrix}$$

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