

# ELASTIC SYSTEM DESIGNED FOR BASE INSULATION AGAINST DYNAMIC ACTION OF VIBRATIONS AND SEISM

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## ABSTRACT

*The approach regarding building base insulation puts into evidence the possibility to evaluate the antiseismic insulation degree in a direction defined using supporting elastic systems having various geometry, structure and antivibrating materials. The antiseismic systems structure is based on the convenient assembly of some elastic and dissipating individual elements so that for rated spectral composition of the earthquake the resonance dynamic response could be avoided. The paper presents three basic technical solutions, Romanian patented, able to provide very low frequencies and allowing low frequency tuning for the building- supporting elements system.*

### 1. General

The approach in the field of building base insulation (for the entire building, some of its parts or equipment) puts into evidence the possibility to evaluate the antiseismic insulation degree in a direction defined using supporting elastic systems having various geometry, structure and antivibrating materials. In fact, all these systems experimentally constructed and used are characterized by elastic performances corresponding to attain the concerned eigen frequencies in the dynamic freedom degrees directions as well as by damping performances so that the internal energy dissipation reduces significantly the shock in the earthquake acting direction.

The antiseismic systems structure is based on the convenient assembly of some elastic and dissipating individual elements so that for rated spectral composition of the earthquake the resonance dynamic response could be avoided.

This paper puts into evidence three technical solutions for elastic systems, patented in Romania, that are able to attain very low eigen frequencies performing low tuning for the system consisting of building-supporting system.

In the first approximation, the hypothesis that the dynamic system building-elastic support has only three dynamic freedom degrees, in particular the three orthogonal translations, has been adopted.

### 2. The Analysis Of The Elastic Supporting Solution Consisting Of Antivibrating Rubber Elements

The supporting classic solution represents a directly elastic system SED consisting of rubber supporting groups located between the building body and the ground embedded concrete foundation.

Figure 1 illustrates this model where the possibility to obtain three translations in the fixed reference axis direction  $Ox$ ,  $Oy$  and  $Oz$  has been provided. For a single rubber support the solution is to construct a block consisting of three identical rubber elements having metallic intermediate reinforcement, an elastic element having  $100 \times 100 \times 25$  cm, antivibrating rubber with  $70^\circ \text{ShA}$  hardness,  $E_s = 15 \text{ MN/m}^2$  static longitudinal elastic modulus,  $G_s = 1,6 \text{ MN/m}^2$  static transversal elastic modulus and  $\Phi = 1$  the shape factor.

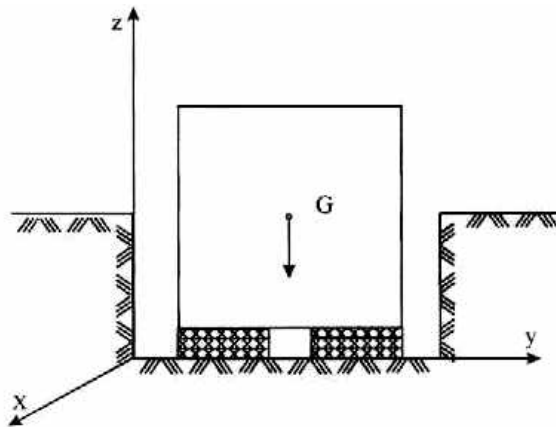


Figure 1. Classic supporting solution

In case of a single element 100x100x25 cm the compression rigidity coefficient  $k_x = k_0$  is  $k_0 = 180 \text{ MN/m}$ , in case of an elastic support  $k_0 = 180 \text{ MN/m}$  and in case of a sandwich elastic support  $k_1^c = \frac{1}{3}k_0 = 60 \text{ MN/m}$ .

The shearing rigidity coefficient for a single element (100x100x25 cm) is  $k_x = k_y = 3k_0^f = 6,4 \text{ MN/m}$  and for a sandwich elastic support is  $k_1^f = 2,13 \text{ MN/m}$ .

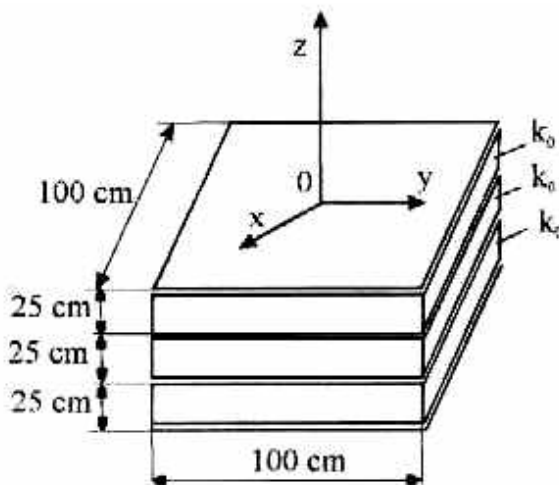


Figure 2. Sandwich element

In particular, in case of a building having 3000t weight and  $G=30 \text{ MN}$  supported by 50 sandwich elements the following eigen frequencies have been obtained:

- ∅ in the horizontal directions  $f_x=f_y=0,94 \text{ Hz}$ ,  $T_x=1,06 \text{ s}$ ;
- ∅ in the vertical direction  $f_z= 5,0 \text{ Hz}$ ,  $T_x=0,2 \text{ s}$ .

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### 3. The Analysis Of The Simple Antiseismic System ASS

The simple antiseismic supporting elastic system can be represented as an elastic unit consisting of a beam with the length  $L$ , its elastic connections being located at its ends and in an intermediate point dividing the length  $L$  into  $a$  and  $b$  (see Figure 3). This beam has two fixed points  $A$  and  $B$  and the moving point  $C$  in connection with the building. By using more elastic units ASS, an elastic system supporting the building has been obtained. This system provides attaining eigen frequencies lower than in previous case using the same number of rubber elements (100x100x25 cm) arranged in a structure specific to the system and aving the same number of supporting units  $N = 15$  pieces. In case of a single elastic unit ASS, the vertical rigidity coefficient is:

$$k_z = k_1^c = \frac{k_0^c}{2(\lambda^2 + \lambda + 1)}$$

where

$$k_0^2 = 180 \text{ MN/m}$$

for a single rubber element 100x100x25 cm;

$$\lambda = \frac{b}{a}$$

In case of the entire system consisting of  $N$  groups ASS the vertical equivalent rigidity coefficient is:

$$k_z^{system} = N \cdot k_1^c$$

For  $N=50$  pieces and  $\lambda = 4$  we have:

$$k_z^{system} = 50 \frac{180}{2(16 + 4 + 1)} = 214,28 \text{ MN/m}$$

- ∅ the eigen frequency  $f_z=1,33 \text{ Hz}$
- ∅ the eigen period  $T_z=0,748 \text{ s}$
- ∅ the static deflection  $\Delta_{st}=0,14 \text{ m}$ .

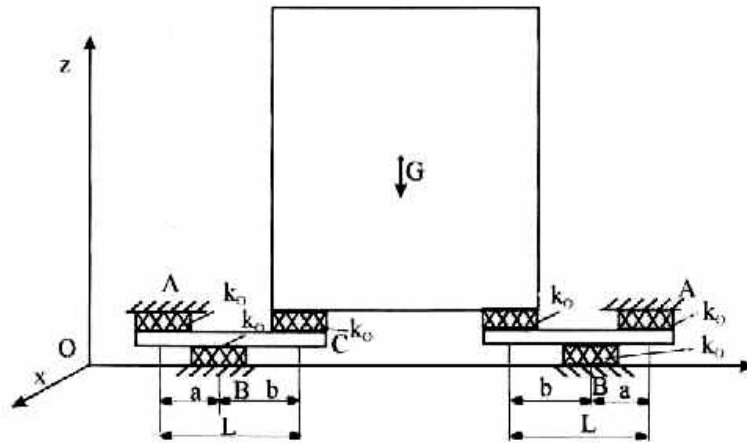


Figure 3. Simple antiseismic elastic supporting system ASS

The horizontal equivalent rigidity coefficient for the whole system consisting of  $N=50$  elastic groups ASS is:

$$k_x = k_y = 3k_0^f N$$

or

$$k_x = k_y = 3 \cdot 6,4 \cdot 50 = 960 \text{ MN/m}$$

Thus, the eigen vibration parameter values in the horizontal direction will be:

- $\omega$  the eigen frequency  $f_x=f_y=2,84$  Hz;
- $\tau$  the eigen period  $T_x=T_y=0,35$  s.

#### 4. The Analysis Of The Composed Antiseismic System ASC

The composed antiseismic elastic system under the form of one individual unit consists of two beams elastic connected by means of rubber elements so that by effects superposing a maximum displacement of the point D connected with the building has been obtained (see Figure 4).

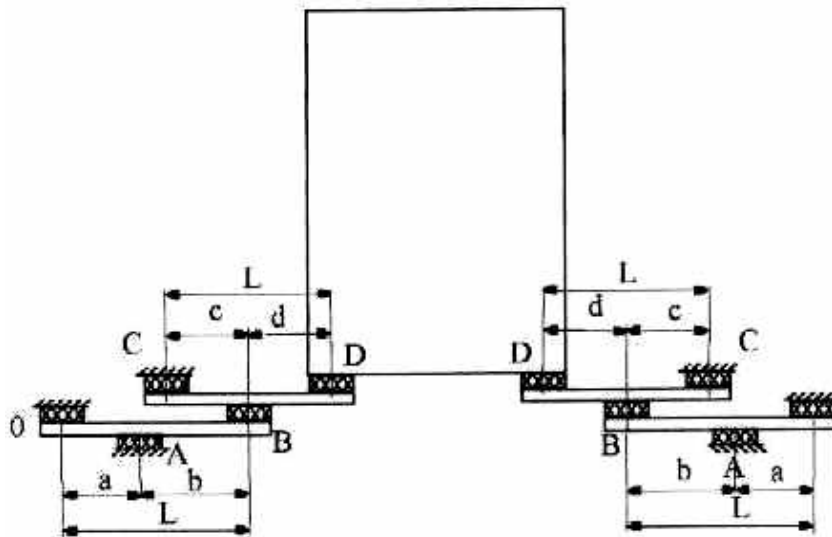


Figure 4. Composed elastic antiseismic system

In case of a single composed unit ASC, the vertical equivalent rigidity coefficient can be written under the form:

$$\frac{1}{k_1^{ASC}} = \frac{1}{\frac{1}{k_0} + \frac{1}{k_0} \left(\frac{d}{c}\right)^2 + \frac{1}{k_1^{OAB}} \left(\frac{c+d}{c}\right)^2}$$

with

$$k_1^{OAB} = \frac{k_0}{2(\lambda^2 + \lambda + 1)}$$

and using notation  $\mu = \frac{d}{c}$  we have:

$$k_1^{ASC} = \frac{k_0}{2(\lambda^2 + \lambda + 1)(\mu^2 + \mu + 1) + \mu^2 + 1}$$

For  $\lambda = 2$  and  $\mu = 2$  it results in:

$$k_1^{ASC} = \frac{180}{2(4+2+1)(4+2+1)+4+1} \cong 1,4MN/m$$

and for the system consisting of N=50 units we have  $k_{z\ system}^{ASC} = 50 \cdot 1,4 = 70MN/m$  with the eigen frequency for the eigen vibration in the vertical direction  $f_z = 0,76Hz$  and  $T_z = 1,31$  s. In case of a simple unit ASC the horizontal equivalent rigidity coefficient is:

$$k_{1x}^{ASC} = \frac{4}{7}k_0^f$$

and for the whole system consisting of N=50 units:

$$k_{xASC}^{system} = 50 \frac{4}{7}k_0^f$$

or

$$k_{xASC}^{system} = 50 \frac{4}{7} \cdot 6,4 = 182,85MN/m.$$

The eigen vibrations characteristics in the horizontal direction are the following:

- the eigen frequency  $f_x = 1,24$  Hz;
- the eigen period  $T_x = 0,8$  s.

By analysing these three elastic supporting solutions it results in the following parameters:

Table 1.

No	Parameter / Solution	Vertical direction		Horizontal direction	
		$f_z,$ Hz	$T_z,$ s	$f_x=f_y,$ Hz	$T_x=T_y,$ s
1	SED	5,00	0,200	0,94	1,06
2	ASS	1,33	0,748	2,84	0,35
3	ASC	0,76	1,310	1,24	0,8

We notice that in case of SED system the lowest eigen frequency in the horizontal direction and in case of ASC system the lowest eigen frequency in the vertical direction have been obtained.

## 5. Antiseismic Insulation Performances

### a) Directly supported elastic system –SED

The equipment static deflection under its dead weight  $G = 10^4N$  is:

$$\Delta_{st} = \frac{G}{k_0N_1}$$

and the fundamental eigen frequency can be calculated as:

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{k_0N_1}{m}}$$

or

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{g}{\Delta_{st}}}$$

or if one replaces the primary data he obtains  $\Delta_{st} = 7,3 \cdot 10^{-3}m$  and  $f_0 = 5,81Hz$ .

Taking into account the equipment has the working frequency, the antivibrating insulation degree I is given by relation:

$$I = 1 - \left| \frac{1}{1 - \Omega^2} \right|$$

where  $\Omega = \frac{f}{f_0}$  represents the relative frequency.

Thus, for SED system, one obtains the insulation degree unsatisfactory for the imposed condition.

### b) Simple elastic system –SES

The vertical equivalent rigidity factor is:

$$k_{ech}^{SES} = \frac{k_0N_2}{2(\lambda^2 + \lambda + 1)}$$

with  $\lambda = \frac{a}{b}$  as factor designed to check the deformation multiplying effect for the first elastic step.

The fundamental eigen frequency is:

$$f_0^{SES} = \frac{1}{2\pi} \sqrt{\frac{k_0N_2}{2(\lambda^2 + \lambda + 1)}}$$

and the frequency ratio  $v = \frac{f_{SES}}{f_0}$  results under the form:

$$v = \frac{f_{SES}}{f_0} \sqrt{\frac{N_2}{2N_1} \frac{1}{\lambda^2 + \lambda + 1}}$$

Replacing  $N_1 = 8$  and  $N_2 = 12$ , we have:

$$v = 0,866 \frac{1}{\lambda^2 + \lambda + 1}$$

The vibration insulation degree due to SES is as follows:

$$I^{SES} = 1 - \left| \frac{1}{1 - \Omega_{SES}^2} \right|$$

with  $\Omega_{SES} = \frac{f}{f_{SES}} = \frac{f}{f_0 v}$  and  $\Omega_{SES}^2 = \frac{4,62}{v^2}$  so we have:

$$I^{SES} = 1 - \left| \frac{1}{1 - \frac{4,62}{v^2}} \right|$$

Figure 5 illustrates the graphic presentation of the eigen frequency ratio  $v$  as well as the insulation degree  $I^{SES}$  as a function of  $\lambda$ .

**c) Composed elastic system-SEC**

The vertical equivalent rigidity factor is:

$$k_{ech}^{SEC} = \frac{k_0 N_3}{2(\lambda^2 + \lambda + 1)(\mu^2 + 2\mu + 1) + \mu^2 + 1}$$

with  $\mu = \frac{d}{c}$  represents the factor designed to check the deformation multiplying effect for the second elastic step.

The fundamental eigen frequency is expressed by:

$$f_0^{SES} = \frac{1}{2\pi} \sqrt{\frac{k_0 N_3}{m[2(\lambda^2 + \lambda + 1)(\mu^2 + 2\mu + 1) + \mu^2 + 1]}}$$

and the frequency ratio results in:

$$\varphi = \frac{f_0^{SES}}{f_0} = \sqrt{\frac{N_3}{N_1} \frac{1}{2(\lambda^2 + \lambda + 1)(\mu^2 + 2\mu + 1) + \mu^2 + 1}}$$

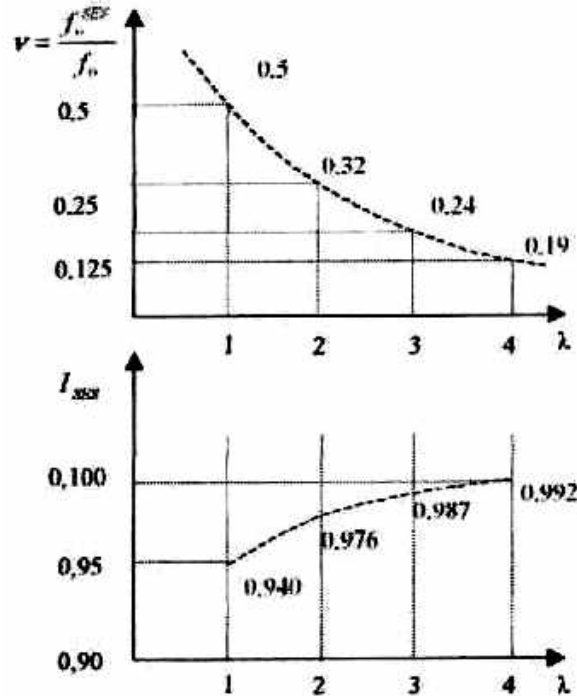


Figure 5. Eigen frequency ratio  $v$  and insulation degree  $I^{SES}$

Replacing  $N_1 = 18$  pieces and  $N_3 = 20$  we obtain:

$$\varphi = 1,58 \frac{1}{\sqrt{8(\lambda^2 + \lambda + 1)(\mu^2 + 2\mu + 1) + \mu^2 + 1}}$$

Figure 6 illustrates the curve family  $\varphi = f(\lambda, \mu)$  having as parameters  $\mu = 1, 2, 3, 4$ .

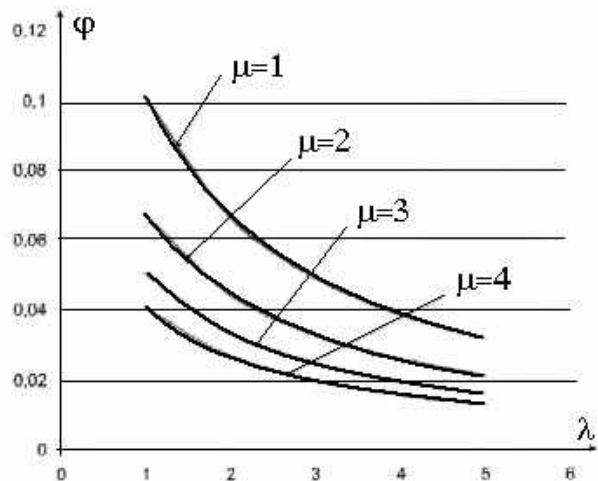


Figure 6. Curve family  $\varphi = f(\lambda, \mu)$

Figure 7 presents the insulation degree curve family  $I^{SEC}$  as a function of  $\lambda = 1, 2, 3, 4$  for  $\mu = 1, 2, 3, 4$ .

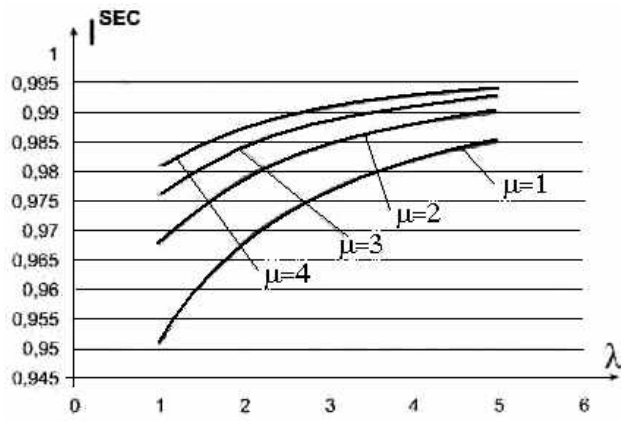


Figure 7. Insulation degree curve family  $I^{SEC}$

The implementation of elastic steps designed to multiply the static deflection under loading, leads the fundamental eigen frequency to attain very low values as well as the possibility of an efficient antivibrating insulation having values  $I > 0,94$ .

The system design has to be done in order to meet all the resistance and stability requirements for both the elastic elements and the whole system.

### References

- [1] Mihăilescu Șt., Bratu P. *Mașini de construcții.volumul 2*, București, Editura Tehnic, 1985.
- [2] Bratu P. *Vibrațiile sistemelor elastice*, Editura Tehnica, București, 2000.