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SOME ASPECTS OF THE CALCULATION OF BEAMS LOADED WITH MOBILE FORCES WITH VARIABLE INTENSITY

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ABSTRACT

In this article is presented the case of the beams which are loaded by mobile forces with variable intensity along the beam. In this context, after calculus assumptions and calculus model are presented, necessary formulas are derived for the analytical study of such beams.
In the final part of the article is presented a case study and some recommendations and conclusions.

KEYWORDS: beams, mobile forces, variable intensity

1. Preliminary

In treaties for strength calculating of the beam structures, is presented the case when the beams are loaded with mobile forces that have constant intensity throughout the movement.
In contrast, a case quite common in practice is that the beams are loaded with mobile forces with variable intensity along the beam. This situation is met if a filling material (sand, gravel, etc.) is deposited on a structure supported on beams.
Assuming that the unloading device ensures a constant flow, the layer height of filling material may be constant or variable along the path that are filed, as the vehicle speed is constant or not. In fig. 1 is presented, schematically, a practical situation that is appropriate to the case described above.

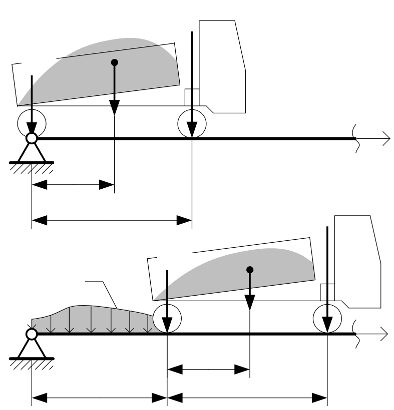

Fig. 1 - Creating a layer of gravel

The notations that were used in fig. 1 have the following meanings:
G_1, G_2 – the equipment weight fractions (excluding the weight of the filling material); those are transmitted to the beam through the rear axle and front axle, respectively;
$G_m(x)$ – function describing the variation of the filling material weight along the beam;
$p(x)$ – function that describes the variation of the distributed load along the beam;
a – the distance between axles;
$d(x)$ – function describing the change of distance between G_1 and G_m forces.
In addition, unlike the classic case, as shown in fig. 1, while forces moving on the beam, acting and a distributed force, which is caused by the weight of the filling material.
In the following part of the paper we present the necessary calculus elements for the study of such practical situations.

2. Assumptions, calculus model, formulas

First, we assume that the speed of the equipment along the beam is low enough so that the forces acting on the beam can be considered as static forces.
If G is the total weight of equipment (excluding the weight of the filling material), the fractions G_1 and G_2 can be expressed by

$$G_1 = (1-k)\cdot G;\; G_2 = k\cdot G \tag{1}$$

where k is a positive factor subunit.

On similar terms, the function describing the variation of the filling material weight along the beam is

$$G_m(x) = [\lambda_0 - \lambda(x)]\cdot G \tag{2}$$

where $\lambda(x)$ is the discharging function and λ_0 the mean proportion of weight equipment that is the weight of the filling material, at the process beginning. On similar terms, let be λ_3 proportion of weight equipment that is the weight of the filling material, at the process ending. Under these conditions, the function $\lambda(x)$ is a monotone increasing positive function with zero as minimum value and $\lambda_0 - \lambda_3$ as maximum value, as seen in fig. 2.

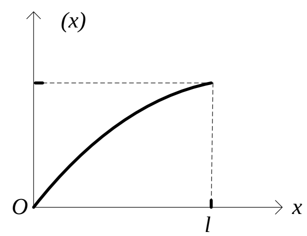

Fig. 2 - The discharging function

The function $d(x)$ describing the change of distance between G_1 and G_m forces, along the beam can be expressed in terms of distance between axles

$$d(x) = a\cdot\beta(x) \tag{3}$$

where $\beta(x)$ is a positive subunit-valued function. It should be pointed out that if attachment machinery is maintained in the same rotated position during the whole process, then the function $\beta(x)$ is a quasi-constant function: $\beta(x) \cong \beta = ct.$

The distributed load intensity depends on the discharging function $\lambda(x)$.

Taking into account the expression 1, it is easily shown that function $p(x)$ has the following form

$$p(x) = G\cdot\frac{d\lambda}{dx} \tag{4}$$

$\lambda_0 - \lambda_3$

The variable force $G_m(x)$ is transmitted to beam through the equipment axles. Therefore, in addition to the constant forces G_1 and G_2, the beam is loaded with two variable forces, $Q_1(x)$ and $Q_2(x)$, that are obtained using the decomposition of a force in two parallel directions:

$$Q_1(x) = [\lambda_0 - \lambda(x)]\cdot[1-\beta(x)]\cdot G \tag{5}$$

$$Q_2(x) = [\lambda_0 - \lambda(x)]\cdot\beta(x)\cdot G \tag{6}$$

Consequently, the beam is loaded by two variable forces $P_1(x)$ and $P_2(x)$ with same action line as the forces G_1, respectively G_2:

$$\begin{aligned} P_1(x) &= G_1 + Q_1(x) = \\ &= \{1-k+[\lambda_0 - \lambda(x)]\cdot[1-\beta(x)]\}\cdot G \end{aligned} \tag{7}$$

$$\begin{aligned} P_2(x) &= G_2 + Q_2(x) = \\ &= \{k+[\lambda_0 - \lambda(x)]\cdot\beta(x)\}\cdot G \end{aligned} \tag{8}$$

Based on the above, the calculus model is given in fig. 3. Besides the known notation, in fig. 3 were used the following notations:

l - beam length;

$V_0(x)$, $V_3(x)$ - bearing reactions;

$R(x)$ - resultant force of the distributed load;

$r(x)$ - abscissa of the resultant force, $R(x)$.

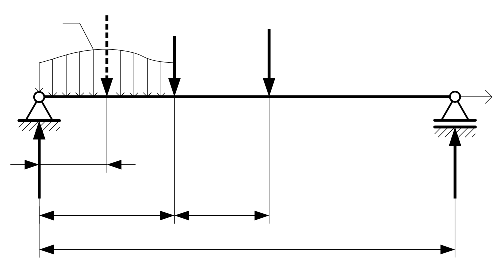

Fig. 3 - The calculus model

Noting with u abscissa associated with a certain section of the beam, the bending moment produced in this section shall be calculated in two different cases, as follows:

1. $x \in [0, l-a]$ - *Both concentrated forces $P_1(x)$ and $P_2(x)$ acting on the beam.*

The bearing reactions is

$$\begin{aligned} V_0(x) &= \left[1-\frac{r(x)}{l}\right]\cdot R(x) + \left(1-\frac{x}{l}\right)\cdot P_1(x) + \\ &+ \left(1-\frac{x+a}{l}\right)\cdot P_2(x) \end{aligned} \tag{9}$$

$$V_3(x) = R(x) + P_1(x) + P_2(x) - V_0(x) \tag{10}$$

The shearing force and the bending moment are multiple-valued functions, the beam having three regions.

- For $u \in [0,x]$:

$$T_1(x) = V_0(x) - R(u) \quad (11)$$

and

$$M_1(u) = u \cdot V_0(x) - [u - r(u)] \cdot R(u) \quad (13)$$

In the formulas (11) and (12), $R(u)$ and $r(u)$ are resultant force of the distributed load corresponding to the distance u, respectively abscissa of this resultant force.

- For $u \in [x, x+a]$:

$$T_1(x) = V_0(x) - R(x) - P_1(x) \quad (14)$$

$$M_1(u) = u \cdot V_0(x) - [u - r(x)] \cdot R(x) - (u - x) \cdot P_1(x) \quad (15)$$

- For $u \in [x+a, l]$:

$$T_1(x) = V_0(x) - R(x) - P_1(x) - P_2(x) \quad (16)$$

$$M_1(u) = u \cdot V_0(x) - [u - r(x)] \cdot R(x) - (u - x) \cdot P_1(x) - P_2 \cdot (u - x - a) \quad (17)$$

where $R(u)$ and $r(u)$ have the meanings that have been described above and $V_0(x)$ is calculated with formula (14).

2. $x \in [l-a, l]$ - *The only concentrated force acting on the beam is force* $P_1(x)$.

Therefore, the calculation of the bearing reactions is done by:

$$V_0(x) = \left[1 - \frac{r(x)}{l}\right] \cdot R(x) + \left(1 - \frac{x}{l}\right) \cdot P_1(x) \quad (18)$$

$$V_3(x) = R(x) + P_1(x) - V_0(x) \quad (19)$$

The shearing force and bending moment are multiple-valued functions, the beam having two regions.

- For $u \in [0,x]$:

$$T_2(x) = V_0(x) - R(u) \quad (20)$$

$$M_2(u) = u \cdot V_0(x) - [u - r(u)] \cdot R(u) \quad (21)$$

where $R(u)$ and $r(u)$ have the meanings that have been described above.

- For $u \in [x,l]$:

$$T_2(x) = V_0(x) - R(x) - P_1(x) \quad (22)$$

$$M_2(u) = u \cdot V_0(x) - [u - r(x)] \cdot R(x) - (u - x) \cdot P_1(x) \quad (23)$$

Let x_{1max} be the abscissa of the beam section where the maximum bending moment occurs in the first case and let M_{1max} be the value of this maximum bending moment. On similar terms, we have x_{2max} and M_{2max} in the second case.

With these notations, the largest maximum bending moment is given by

$$M_{max\,max} = max\{M_{1max}, M_{2max}\} \quad (24)$$

and the abscissa for the corresponding section is given by

$$x_{max\,max} = \begin{cases} x_{1max}\ ; M_{max\,max} = M_{1max} \\ x_{2max}\ ; M_{max\,max} = M_{2max} \end{cases} \quad (25)$$

3. Case study

The case study is built on the general model presented above, on which there were made the following customizations:

- Distributed load along the beam is a uniformly distributed load:

$$p(x) = p = ct. \quad (26)$$

- All of the filling material is deposited on the structure:

$$\lambda_3 = 0 \quad (27)$$

The calculus model is given in fig. 4.

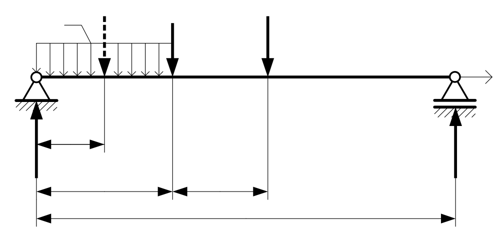

Fig. 4 - The calculus model used by case study

First, considering the relations (4) and (26), we obtain that the function $\lambda(x)$ is a linear function and, consequently, has the form:

$$\lambda(x) = A \cdot x + B \quad (28)$$

The constants A and B are obtained by imposing the boundary conditions for $\lambda(x)$. After calculations, we obtain:

$$\lambda(x)=\lambda_0 \cdot x/l \tag{29}$$

Therefore, on basis of the formulas (4) and (29), we obtain:

$$p=G\cdot\lambda_0/l \tag{30}$$

Under these conditions, the forces $P_1(x)$ and $P_2(x)$ can be calculated with the following formulas:

$$P_1(x)=\left[1-k+\lambda_0\cdot(1-\beta)\cdot\left(1-\frac{x}{l}\right)\right]\cdot G \tag{31}$$

$$P_2(x)=\left[k+\lambda_0\cdot\beta\cdot\left(1-\frac{x}{l}\right)\right]\cdot G \tag{32}$$

The resultant force of the distributed load is given by

$$R(x)=p\cdot x=G\cdot\lambda_0\cdot\frac{x}{l} \tag{33}$$

And abscissa of the resultant force, $r(x)$ is

$$r(x)=\frac{x}{2} \tag{34}$$

If forces $P_1(x)$ and $P_2(x)$ are acting on the structure (the first case treated in the previous paragraph), the bearing reaction $V_0(x)$ is calculated with following formula:

$$V_0(x)= \\ =\left\{\begin{array}{l}\lambda_0\frac{x}{l}\left(1-\frac{x}{2l}\right)+ \\ \left(1-\frac{x}{l}\right)\left[1-k+\lambda_0(1-\beta)\left(1-\frac{x}{l}\right)\right]+ \\ \left(1-\frac{x+a}{l}\right)\left[k+\lambda_0\beta\left(1-\frac{x}{l}\right)\right]\end{array}\right\}\cdot G \tag{35}$$

If the only concentrated force acting on the beam is force $P_1(x)$, the bearing reaction $V_0(x)$ is calculated with following formula:

$$V_0(x)= \\ =\left\{\begin{array}{l}\lambda_0\frac{x}{l}\left(1-\frac{x}{2l}\right)+ \\ \left(1-\frac{x}{l}\right)\left[1-k+\lambda_0(1-\beta)\left(1-\frac{x}{l}\right)\right]\end{array}\right\}\cdot G \tag{36}$$

Based on force $V_0(x)$, can be determined in each case, the formulas which describe the variation of the shearing force and the bending moment, along the beam.

Consequently, it is possible to be locate the dangerous section of the beam $x_{max\,max}$ and to be calculate the largest maximum bending moment, $M_{max\,max}$.

4. Conclusions

Correct sizing of beams which are loaded with mobile forces with variable intensity is a problem that requires a detailed study using a more complicated calculus model, than the calculus model used for beams which are loaded with constant moving loads.

In this context, the calculus model proposed in the paper is likely to lead to getting results which are characterized by a better accuracy than the one obtained on basis of simplified calculation models.

It is recommendable to use the model proposed in the paper, especially in cases where the weight of filling material is of the same order of magnitude as the equipment weight and a is at least 10% of l.

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