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MUFFLER PERFORMANCES MODELING WITH BOUNDARY ELEMENT - METHOD AND EXPERIMENTAL STUDY

Assist. Prof. Dr. Eng. Ovidiu VASILE
Department of Mechanics
POLITEHNICA University of Bucharest
Assoc. Prof. Dr. Eng. Nicusor DRAGAN
MECMET - The Research Center of Machines, Mechanic and Technological Equipments
"Dunarea de Jos" University of Galati

ABSTRACT

This paper investigates the acoustic performance of a reactive silencer using Boundary Element Method analysis and experimental techniques. This analysis addresses a research topic of major interest today on how to reduce noise pollution due to vehicles, or pressure vessels of various machinery and equipment involving air ejection as a result of various processes or urban expansion.

Modeling procedures for accurate performance prediction had led to the development of new methods for practical muffler components in design. The transmission loss is the more widely used and can be easily computed with a Boundary Element Method analysis. In the present paper the author present an overview of the principles of Boundary Element Method for predicting the transmission loss (TL) of a muffler with two expansion chambers, the pressure distribution on surfaces of the muffler and the principles are compared with the acoustic performances of the experimental study set up. The predicted results agreed in some limits with the experimental data published in literature.

KEYWORDS: boundary element method, muffler, transmission loss

1. THEORETICAL CONSIDERATIONS ON MUFFLER PERFORMANCES

The most widely used performance used to characterize mufflers is surely the transmission loss (*TL*), other indexes are however available such as insertion loss (*IL*) and noise reduction (*NR*), and a good understanding of the differences among them is fundamental in order to apply the most appropriate to each situation. Considering a generic muffler or duct as depicted in fig. 1, we assume that the pressure p_1 at the inlet is composed by two waves one traveling towards right (entering the muffler) that is called p_1^+ , and the other traveling in the opposite direction and called p_1^-.

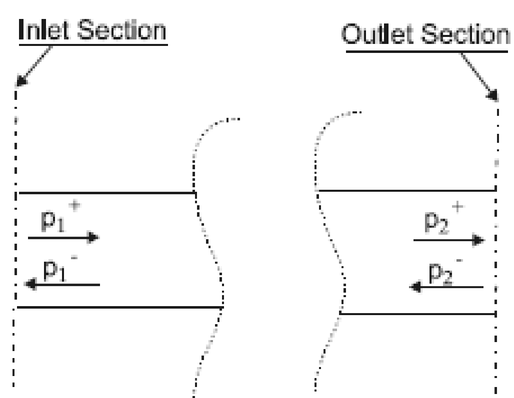

Figure 1 The inlet and outlet of muffler or duct

At the outlet the situation is similar and the total pressure p_2 is composed by two waves travelling in opposite directions. The velocity at the inlet (V_1) and outlet (V_2) sections can also be expressed in terms of the two components of the waves. The overall relations are:

$$p_1 = p_1^+ + p_1^- \quad (1)$$

$$V_1 = (1/rho \cdot c) \cdot \left(p_1^+ - p_1^-\right) \quad (2)$$

$$p_2 = p_2^+ + p_2^- \quad (3)$$

$$V_2 = (1/rho \cdot c) \cdot \left(p_2^+ - p_2^-\right) \quad (4)$$

where rho is the air density

The *TL* is defined as the ratio between the sound power that actually enters in the muffler and the transmitted sound power. The sound power that enters the muffler is associated to the right travelling wave at the inlet (p_1^+), while the transmitted sound power is associated to the right travelling wave at the outlet (p_2^+). In other words the *TL* is the ratio $(p_1^+)^2/(p_2^+)^2$.

The transmission loss is more widely used mainly because it can be more easily evaluated theoretically since it is an intrinsic property of the muffler, while the Insertion loss depends instead on the acoustic impedance at the inlet and outlet. If the impedance at inlet and outlet are both equal to the fluid impedance, then the insertion loss is equal to the transmission loss.

2. EVALUATION OF TRANSMISSION LOSS

Table 1 Boundary conditions

Set	Boundary condition	
	at inlet	at outlet
1	Imposed velocity v=1	Imposed velocity v=0
2	Imposed velocity v=1	Imposed pressure p=0

The standard procedure for evaluation of *TL* is based on the evaluation of the so-called four pole parameters (*A*, *B*, *C*, *D*) that characterize the muffler. In the past, several studies were conducted in order to analytically evaluate these parameters, but nowadays they can be easily computed with a BEM analysis. It is simply required to execute two sets of calculations that differ only for the boundary conditions applied at the outlet. The calculations to be performed are respecting Table 1.

The four parameters (that are complex numbers that depend on frequency) can then be computed as:

$$\text{from set 1: } A = \left(p_1 / p_2\right) \quad (5)$$

$$\text{from set 2: } B = \left(p_1 / v_2\right) \quad (6)$$

$$\text{from set 1: } C = \left(v_1 / p_2\right) \quad (7)$$

$$\text{from set 2: } D = \left(v_1 / v_2\right) \quad (8)$$

An interesting property of the above parameters is that they satisfy the relation *AD-BC*=1, and this can be used as a useful check for ensuring the accuracy of the performed calculations. Using the equations (1), (2), (3), (4) and the above definitions of *A*, *B*, *C*, *D* it is possible to obtain an expression for the *TL*.

The transmitted pressure p_2^+ can be most easily determined if the outlet is non-reflecting that is if p_2^- =0. Using then equations (1) , (2), (3), (4) and the above definitions the ration $(p_1^+)/(p_2^+)$ can be easily obtained and the transmission loss is writen as:

$$TL = 20 \log_{10} \left[\frac{\left| A + \frac{B}{rho \cdot c} + C(rho \cdot c) + D \right|}{2} \right] \quad (9)$$

3. BOUNDARY CONDITION

A critical issue for an accurate evaluation of *TL* and *IL* is the correct application of Boundary Conditions (BC), in particular in regions where they change acc. to [1], [2], [3].

For the inlet region, we need to apply a constant velocity at the inlet section while in the other nodes of the duct the BC is still an imposed velocity but with zero velocity. The situation is depicted in fig. 2.

We consider the point *P* that is at the intersection of the inlet section with the duct surface. Velocity has to be applied to this point, if we consider this point as belonging to the inlet section we should impose a unitary normal velocity while if we consider it as belonging to the duct we should impose a null velocity. The correct velocity to apply, if we come back to the definition of the velocity BC we remember that this BC consists in ensuring that the fluid velocity in the direction of the perpendicular to the surface is equal to the imposed value. But what is the direction of the surface normal for the point P. Theoretically speaking, the normal is not defined since the surface is not smooth at this point, however practically the surface normal for a generic point is always computed taking the average of the normal of all the panels at which the node is connected.

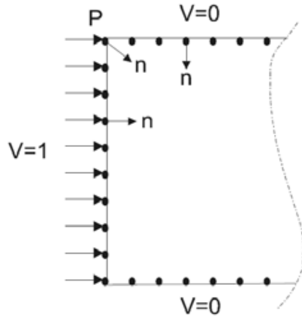

Figure 2 Inlet region

The right solution is the possibility to split the node P in two nodes P_1 and P_2 having the same geometrical coordinates but one connected to the panel of the inlet section and the other connected to the panel of the duct, as depicted in fig. 3. In the picture, the view is exploded and the points P_1 and P_2 are showed at different places but this is only for visualization reason and they should have instead the same geometrical coordinates.

The important thing is however for the node P_1 to be connected only to the panel of the inlet section and the node P_2 to be connected only to the panel of the duct. Now the surface normal for the point P_1 is horizontal since the point is no more connected to any panel of the duct. Reciprocally, the normal of the point P_2 is now vertical. This operation of splitting the nodes is referred to as *disconnection*, since the elements of the inlet are no more topologically connected to the elements of the duct [4], [5], [6], [7], [8].

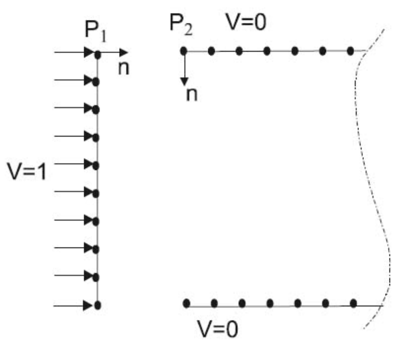

Figure 3 Operation of splitting the nodes

The same kind of problem can appear for example at the outlet. In the point at intersection of outlet and duct, this situation is also more difficult since we have that different kind of BC to be applied to points of duct and outlet, since for the outlet section we need to assign a pressure BC while for the duct we have as usual a Velocity BC. The correct BC to be applied for the point at intersection can now be easily obtained, also in this case we need to introduce two coincident nodes, one connected to the panels of the duct and the other connected to the panel of the outlet and apply the relative BC to each node.

4. CASE STUDY. CONCLUSIONS

As a practical example, we are now going to consider a muffler with two expansion chambers, using the demo version of VNoise software, and we are going to evaluate the *TL*.

The model is defined inserting the nodes that define the profile of the muffler and then is generated a revolution surface from them. The nodes coordinates to be inserted are presented in Table 2. By connecting them with edges, we obtain the base model represented in fig. 4.

Table 2 The nodes coordinates

Node	**Coordinate [m]**		
	x_i	y_i	z_i
N_1	-0.40	0.05	0.00
N_2	-0.40	0.05	0.00
N_3	-0.30	0.20	0.00
N_4	-0.30	0.20	0.00
N_5	0.00	0.05	0.00
N_6	0.00	0.20	0.00
N_7	0.30	0.20	0.00
N_8	0.30	0.05	0.00
N_9	0.40	0.05	0.00
N_{10}	0.40	0.00	0.00

We apply the required BC and then perform the discrimination using 6 points per wave at 4500 Hz. Figure 5 shows the discrimination of the muffler with 216 nodes, for example to generate a 90° revolution surface and using symmetries during calculations.

Figures 6 and 7 show velocity distribution and pressure distribution on surfaces of the muffler with two expansion chambers for 3000 Hz.

In order to evaluate *TL*, we need to execute two sets of calculation, one with v=0 at the outlet and the other with p=0 at the outlet (acc. to Table 1). We consider a calculation in the range of (50-3000 Hz) with a step of 10 Hz, using a rotational symmetry, considering only a ¼ of the muffler. In this case, fig. 8 presents the variation of transmission loss (*TL*) for the muffler with two expansion chambers.

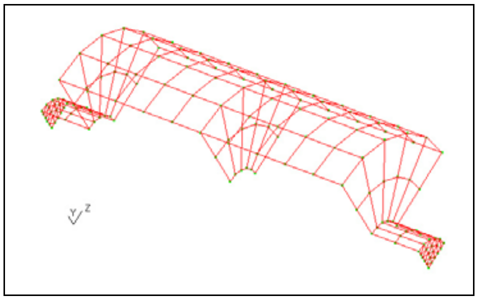

Figure 5 Discrimination of the muffler (216 nodes)

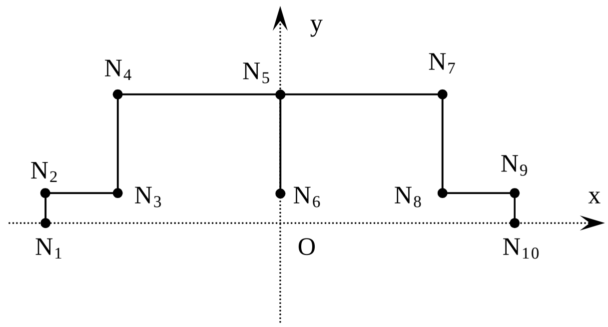

Figure 4 Base model of muffler

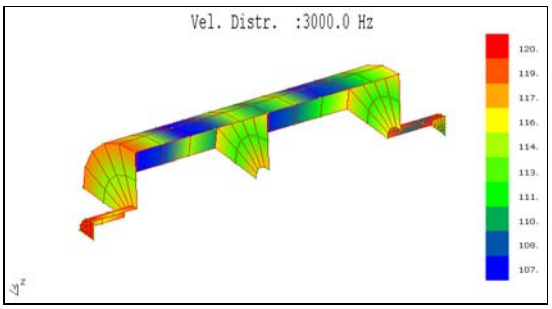

Figure 6 Velocity distribution at 3000 Hz

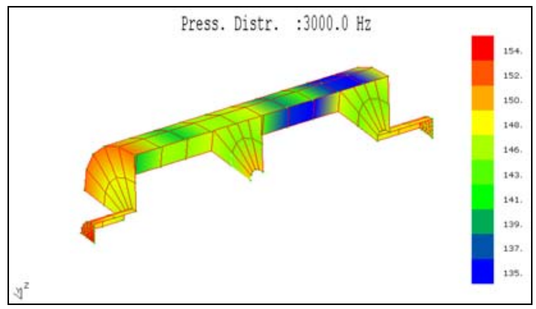

Figure 7 Pressure distribution at 3000 Hz

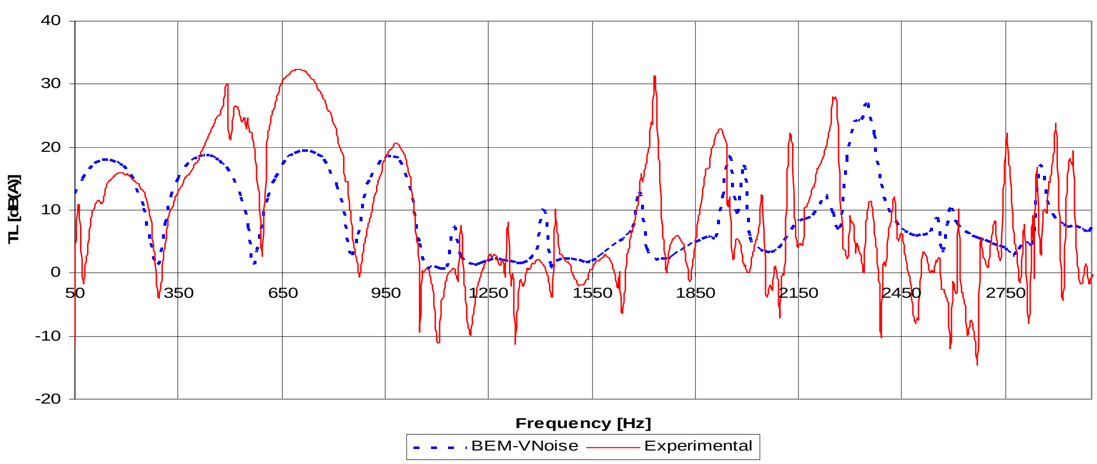

Figure 8 Transmission loss (*TL*) for the muffler with two expansion chambers

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