# DYNAMICAL RESPONSE ANALYSIS OF A SYSTEM WITH ONE DEGREE OF FREEDOM STRESSES BY THE DIFFERENT PULSE EXCITATION FUNCTIONS

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### ABSTRACT

Viscoelastic systems embedded in structures such as bridges are subject to various dynamic loading from traffic. A dynamic study of these structures requires more accurate definition of disturbance function parameters: shape, duration and amplitude. This paper is a theoretical study that analyzes the dynamic response of a system with one degree of freedom with a viscoelastic element embedded (Kelvin rheological model) loaded by various functions of the excitation pulse. In this way it is highlighted the importance of the excitation function shape, in physical and mathematical modeling of technological systems.

KEYWORDS: vibration, viscoelastic, dynamic, rheological

#### **1. Problem formulation**

Let's consider a mechanical system with one degree of freedom stimulus by a pulse excitations F(t). To evaluate the influence of excitation type on the dynamic response function of the mechanical system in fig. 1, were determined the following parameters of development: time displacement of mass m on the Oy direction and frequency response.



The analysis of these parameters was achieved for various forms of dynamic excitation functions of the system, assuming the following numeric values of its parameters:  $k_1=24\cdot10^8$ N/m;  $c_1=7\cdot10^6$  Ns/m;  $m_1=16,2\cdot10^3$  kg:  $F=1000\cdot10^3$  N

# 2. Dynamic response of mechanical system to different pulse stimuli

The dynamic response of considered mechanical system will be analyzed in stimuli with the following pulse excitation hypothesis: rectangular, half-sine, haversine, triangular and trapezoidal.

# **2.1 Dynamic response of mechanical system to rectangular pulse stimuli**

Rectangular waveform is defined by the equation (1):

$$f(t) = \begin{cases} A, \ 0 \le t \le T \\ 0, \ -\infty < t < 0, T < t < \infty \end{cases}$$
(1)

The determination of these parameters of the system response was made considering the loading force of rectangular wave type with 0.03s wide and amplitude of 1000 N. The rectangular waveform with duration T = 0.03 s and amplitude A = 1000N is presented in fig. 2. According to fig. 3, it is observed that the mass m motion is damped; the residual response of the system is in the range 0.57 to 0.8 s. The dominant frequency band, in this case, is in the range 0-33 Hz, fig. 4.



Fig. 2 Rectangular waveform - T=0,03s



Fig. 3 Dynamic response of mechanical system



### 2.2 Dynamic response of mechanical system to half-sine pulse stimulus

Half-sine waveform is defined by the equation (2):

$$F(t) = \begin{cases} A \sin \frac{\pi t}{T}, \ 0 \le t \le T \\ 0, \qquad -\infty < t < 0, \ T < t < +\infty \end{cases}$$
(2)

Half-sine waveform with duration of the T = 0.03 s and amplitude A = 1000N is presented in fig. 5.



Fig. 6 Dynamic response of mechanical system



Fig. 7 Frequency response

### 2.3 Dynamic response of mechanical system to triangle pulse loading

Triangular waveform is defined by equation (3):

$$F(t) = \begin{cases} A \frac{t}{T}, \ 0 \le t \le T \\ 0, \ -\infty < t < 0, T < t < +\infty \end{cases}$$
(3)

Triangular waveform with duration of the T = 0.03 s and amplitude A = 1000N is presented in fig. 8.



Fig. 8 Triangular waveform - T=0,03s



Fig. 9 Dynamic response of mechanical system



The response parameters of the system have been determined considering the loading force of triangular pulse type with 0.03 s width and 1000 N amplitude, fig. 9, 10..

## 2.4 The dynamic response of mechanical system to haversine pulse shape stimulus

Haversine waveform is defined as in equation (4):

$$F(t) = \begin{cases} \frac{A}{2} (1 - \cos \frac{2\pi t}{T}), 0 \le t \le T\\ 0, -\infty < t < 0, T < t < +\infty \end{cases}$$
(4)

Haversine waveform with duration T = 0.03 s and amplitude A = 1000N is represented in fig. 11.



Fig. 11 Haversine pulse - T=0,03s

The evaluation of the response parameters of the system has been determined considering the loading force haversine pulse type with 0.03 s width and 1000 N amplitude, fig. 12, 13.



Fig. 12 Dynamic response of mechanical system



# **2.5 Dynamic response of mechanical** system to trapezoidal pulse stimulus

The trapezoidal waveform is defined by the equation (5)

$$F(t) = \begin{cases} A \cdot \frac{t-a}{b-a}, a \le t \le b \\ A, \quad b < t < c \\ A \cdot \frac{t-d}{c-d}, c \le t \le d \end{cases}$$
(5)

The trapezoidal waveform with duration of the T = 0.03 s and amplitude A = 1000 N is represented in fig. 14.



Fig. 14 Trapezoidal pulse - T=0,03s

The response parameters of the system have been determined considering the loading force of trapezoidal pulse type with 0.03 s width and 1000 N amplitude, fig. 15, 16.



Fig. 15 Dynamic response of mechanical system



#### 3. Conclusion

The summary results of the parameters evaluation of dynamic vibration response of the mechanical system considered are shown in the following table:

Table 1

No	Type of stimulus function	Maximum value of displacement on OY direction [m]	Dominant frequency [Hz]
1	Rectangular	$6,5 \cdot 10^{-4}$	0-33
2	Half - sine	$5,4.10^{-4}$	0-50
3	Triangular	$4,15 \cdot 10^{-4}$	0-40
4	Haversine	$4,73 \cdot 10^{-4}$	0-70
5	Trapezoidal	$5,71 \cdot 10^{-4}$	0-50

The maximum value of displacement of mass m on vertical direction is identified in case of the loading system by a rectangular type force function, explanation of this phenomenon being the existence of a landing time of the stimulus to the value of 1000N.

Note that the dominant frequency response of displacement mass m has lower values for functions characterized by sharp increases, such as rectangular or triangular type applications.

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