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THE INFLUENCE OF NONLINEAR THE ELASTIC CHARACTER OF VISCOELASTIC SYSTEMS ON THE DYNAMIC RESPONSE OF MECHANICAL SYSTEMS

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ABSTRACT

Bridges and viaducts represent vital engineering structures which are component of terrestrial communication lines, therefore reducing their exposure to factor originating from natural or anthropogenic hazards has no optional character, becoming a necessity and a goal of the current modern economy. Combating destructive vibration phenomenon, or in some cases catastrophic, likely to develop in the absence of control techniques, at the running car or during seismic movement of a bridge (viaduct), is achieved by implementing control systems capable to dissipate the energy accumulated in the system, such as viscoelastic dampers. After prolonged use, these antivibration systems under go structural and functional changes due to viscous and elastic link degradation, thus gaining a nonlinear behavior. This paper represents a theoretical study of the nonlinear behavior influence of the dynamic response of mechanical system in impulsive stress regime.

KEYWORDS: bridge, vibration, viscoelastic, dynamic, nonlinear

1. Introduction

Highlighting the influence of viscoelastic systems nonlinearities on the effective functioning of technological system or propagation of vibrations can be achieved in two ways:

- by establishing and analyzing the *influence of individual parameters*, vibration characteristics and machine foundation resting on viscoelastic systems, as follows:
  1. the response in time of kinematics parameters of vibration
  2. the energy dissipated by viscous friction
  3. the motion trajectory
  4. the power spectral density
- by establishing and analyzing *the parameters of global influence*, able to correlate vibration movement of the whole viscoelastic system
  1. transfer function
  2. coherence function

For example, it is considered a theoretical system with two degrees of freedom (fig. 1) loaded by a rectangular pulse train (fig. 2).

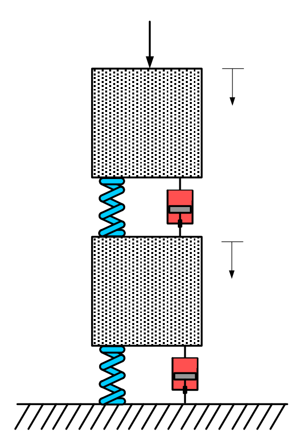


Fig. 1. System with two degrees of freedom

Mass interposed between two viscoelastic systems with the following characteristics: constant damping c_1 and constant elasticity k_1. The ground has viscoelastic behavior characterized by coefficients c_2 and k_2.

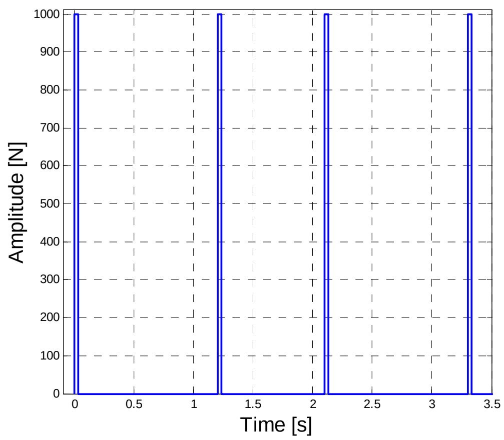


Fig. 2. Excitation signal - four rectangular pulses

Under these assumptions, the system vibration movement is described by the following system of differential equations:

$$\begin{cases} m_1\ddot{x}_1 + c_1\left(\dot{x}_1 - \dot{x}_2\right) + k_1(x_1 - x_2) = F(t) \\ m_2\ddot{x}_2 + c_1\left(\dot{x}_2 - \dot{x}_1\right) + c_2\dot{x}_2 + k_1(x_2 - x_1) + k_2x_2 = 0 \end{cases} \quad (1)$$

Solving this system was carried out using Runge Kutta numerical method of order IV, using the module dedicated to this method of MATLAB software package version 7.0. In order solve the system of differential equations, the following numerical values of mechanical system parameters were considered: $P=2\cdot10^6$ N; $k_1=3\cdot10^9$ N/m; $c_1=5\cdot10^6$ Ns/m; $m_1=134\cdot10^3$ kg; $m_2=10^5$ kg; $k_2=4\cdot10^9$ N/m; $c_2=6\cdot10^6$ Ns/m, **[1]**.

To identify changes induced in the dynamic response system of the nonlinear behavior of the dynamic isolating systems, we will undertake a comparative analysis of *influence parameters* of vibration movement, as follows:

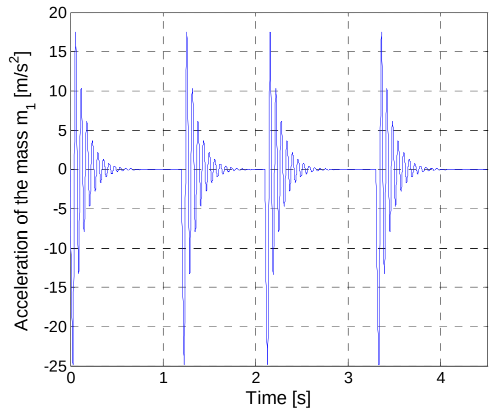
- elastic and damping forces - linear forces
- nonlinear elastic forces F_{el}, linear damping forces F_v

$$F_{el} = k_1(1+\beta x^2)x, \beta = 2\cdot10^8\ 1/m^2 \quad (2)$$

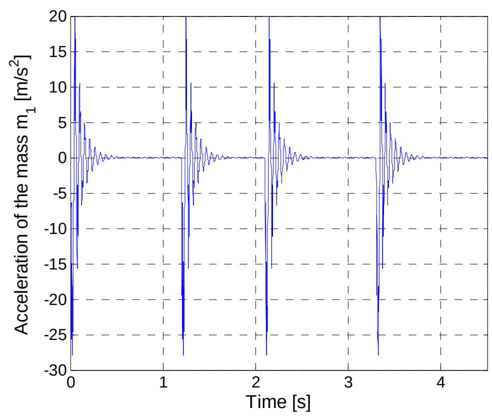
2. Analysis of kinematics vibration parameters

a. The response in time

Developments in time of the kinematics parameters of movement of the masses m_1 and m_2 were analyzed when compared to linear and nonlinear stiffness characteristics of viscoelastic boundary system **[3]**, as in fig. 3a, b and fig. 4 a, b.

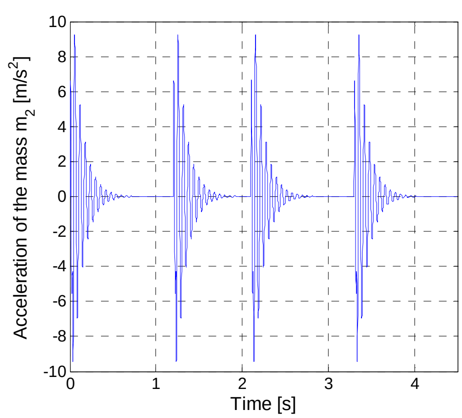


a) F_{el} - linear; F_v - linear

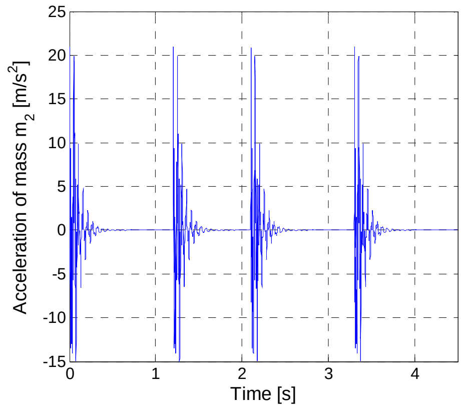


$F_{el}=k_1(1+\beta x^2)x$; F_v - linear

Fig. 3. The acceleration of the mass m_1



a) F_{el} - linear; F_v - linear



b) $F_{el}=k_1(1+\beta x^2)x$; F_v - linear

Fig. 4. The acceleration of the mass m_2

The comparative analysis of these parameters reveals the following key issues:

- mass m_1 acceleration amplitude increases for nonlinear elasticity case;
- mass m_2 acceleration amplitude increases strongly for nonlinear elasticity case;

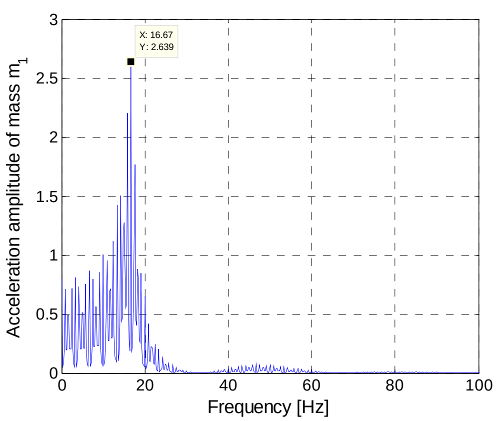
Consequently, for the case of the nonlinear type of elastic forces, the propagated vibration is characterized by a higher acceleration, which increases the harmful degree of sent vibration.

b. The frequency response

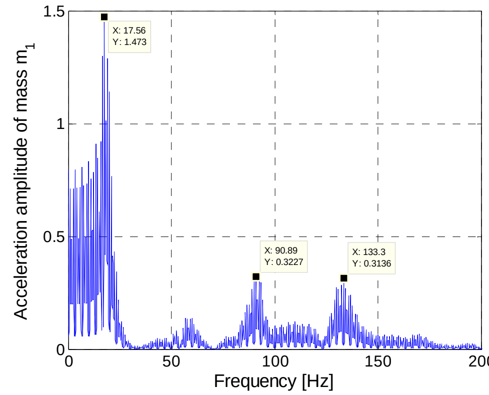
b.1 Spectral representation of the acceleration masses m_1 and m_2

In fig. 5 a, b and 6 a, b are represented in frequency the accelerations of the m_1 and m_2 masses, for cases of linear and nonlinear elastic force of viscoelastic boundary system.

The presence of nonlinearities in the isolation system leads to super-harmonic components in the frequency response of the system.

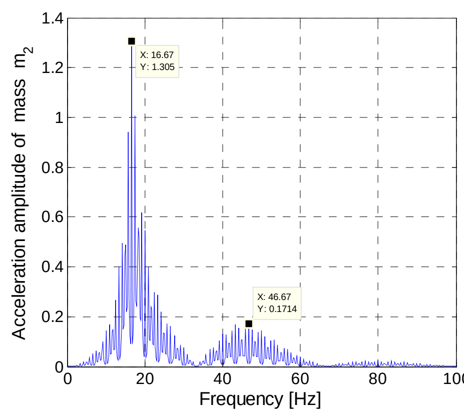


F_{el} - linear; F_v - linear

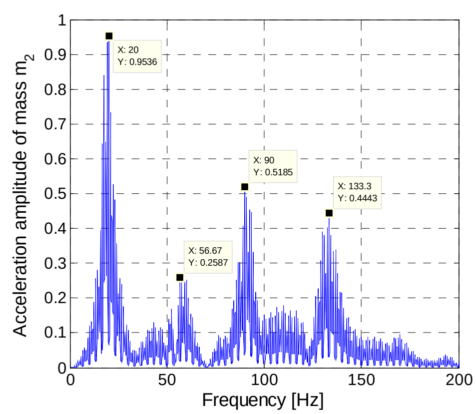


$F_{el}=k_1(1+\beta x^2)x$ F_v - linear

Fig. 5. Spectral representation of the acceleration mass m_1



F_{el} - linear; F_v - linear

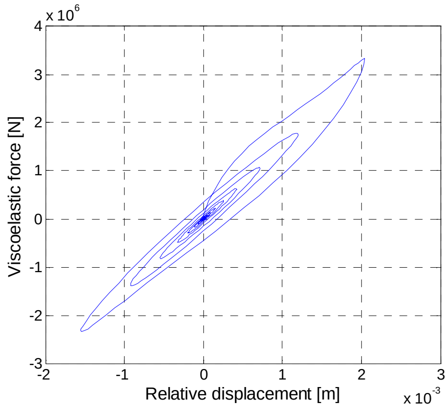


$F_{el}=k_1(1+\beta x^2)x$ F_v - linear

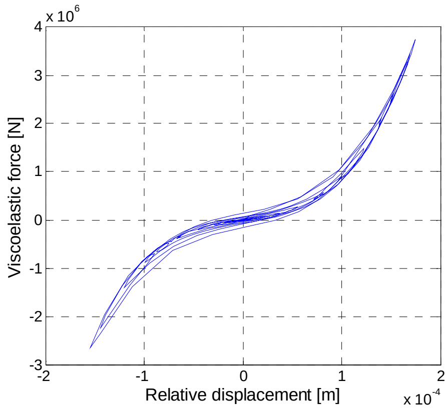
Fig. 6. Spectral representation of the acceleration mass m_2

c. Energy dissipated by viscous friction in a period of movement

The presence of the nonlinear elastic type force in the oscillatory motion of the system causes a clear reduction of energy dissipated by hysteresis, from the value of 1830 J for linear case, to 210 J for nonlinear case, fig. 7.



a. F_{el} - linear; F_v – linear; W=1830J

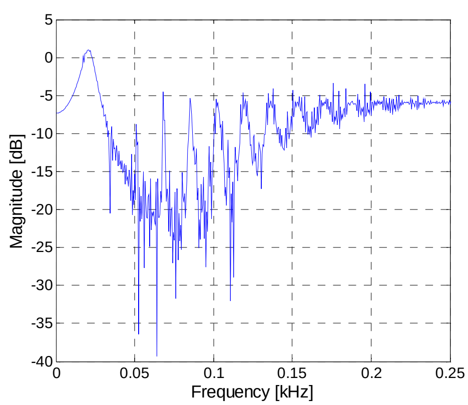


b. $F_{el}=k_1(1+\beta x^2)x$; F_v – linear; W=210J

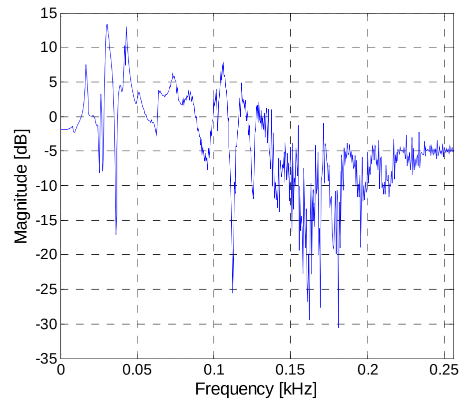
Fig. 7 Energy dissipated by viscous friction in a period of movement

d. The transfer function of power spectral density

To expression of linear the elastic force, this parameter indicates the maximum transfer function at frequency of 17.7 Hz, the same frequency identified as dominant in the acceleration frequency response analysis, fig. 8.



a. F_{el} - linear; F_v - linear



b. $F_{el}=k_1(1+\beta x^2)x$ F_v - linear

Fig. 8 Displacement transfer function

Transfer function representation of motion and speed signals in the case of nonlinear elastic forces show their positive values for different frequency ranges against the linear case which involves a transfer of power signal over a wide frequency band. This change increases the risk of resonance phenomenon for equipment, machinery in the plants vicinity, or may lead to cracks in nearby buildings.

3. Conclusions

Analytical methods were developed for dynamic characterization of the nonlinear vibrations at simple mechanical systems, determining solution relying mostly on approximations. Development of computer technology in recent years has allowed the use of numerical methods to solve mathematical models characteristic for nonlinear dynamic systems. Using numerical methods allowed studying a system with two degrees of freedom which has implemented a passive vibration viscoelastic isolation system characterized by elastic and dissipative nonlinear forces. To quantify the influence of system nonlinearities on the dynamic behavior of viscoelastic mechanical system, the parameters were defined and analyzed for individual and global influence.

Acknowledgement:

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References

[1] **Buzdugan, Gh.,** *Izolarea antivibratorie a maşinilor*, Editura Academiei Române, Bucureşti, 1980.

[2] **Harris, C.M., Piersol A.G.**, *Shock And Vibration Handbook*, McGraw Hill Book Co, 2002.

[3] **Leopa A., Nastac S.,** *Dynamic Behaviour of Foundations in Linear and Nonlinear Elastic Characteristics Hypothesis*, WSEAS Transactions on Applied and Theoretical Mechanics, Issue 4, vol.3, April 2008, ISSN 1991-8747, pp. 145-154.