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CONSIDERTIONS ON THE MECHANICAL VIBRATIONS OF AN ELECTRONIC CIRCUIT BOARD

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ABSTRACT

The paper analyzes the dynamic stresses induced by mechanical vibrations on an electronic circuit board, considering different study hypotheses, resulting from real cases presented in technical texts. The corresponding models are presented and, for one of these models, the diagrams resulted from the experimental study, as well as the vibration eigenforms, are shown and discussed.

KEYWORDS: deformations, stresses, vibrations

1. INTRODUCTION

The paper analyzes the dynamic stresses induced by mechanical vibrations on an electronic circuit board, with the electronic components assembled parallel to the vibration direction (Fig. 1).

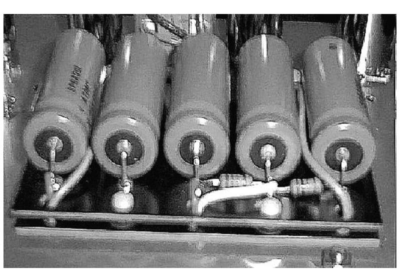

Fig. 1. The electronic circuit board

Due to the high stiffness of the board, its transmissibility is $\tau = 1$. This means that the perturbation is acting directly upon the electronic components, each of them becoming an oscillating system.

Having assumed that the connections are only deformable parts, the model will be reduced to a static undetermined frame.

The system damping may be considered linear for stresses in the elastic domain and highly nonlinear in the elasto-plastic domain, due to the hysteretic phenomena. If the circuit includes massive components with transversal connections (i.e. condensers, quartz oscillators etc.), a rotation tendency occurs, about an axis passing through the mass centre of the component. In this case, two natural frequencies occur (for transversal and torsional vibrations). It can be remarked that the mechanical model is determined by the layout of the component on the board with respect to the direction of the perturbation force, as well as by the mass and the form of the component [4].

2. THE ANALYTIC STUDY

Starting from the cases identified in

practice, the corresponding study models will be analyzed, considering the above mentioned hypotheses.

a) The forces acting upon the electronic components

The force acting upon the component is:

$$F = F_0 \cos \Omega t\,, \tag{1}$$

where the following notations have been used:

F_0 – the amplitude of the perturbation force;

Ω – the circular frequency of the perturbation force;

t – the time.

The stresses in the connectors and connections depend on the force acting upon the electronic component [2],

$$F_b = m \cdot a_b = \frac{m \cdot a_c}{\tau}\,, \tag{2}$$

a_c – the acceleration of the component;

a_b – the acceleration of the board;

τ – the transmissibility of the vibrating system (the ratio between the transmitted force and the acting one).

Formulae (1)-(2) lead to the amplitude of the vibration in the range of:

$$a = \frac{F_0}{m\sqrt{\left(1-\eta^2\right)^2 + 4D^2\eta^2}}\,, \tag{3}$$

where the following parameters have been introduced:

$\eta = \dfrac{\Omega}{\omega}$ – the relative frequency;

$\omega = \sqrt{\dfrac{c}{m}}$ – the natural circular eigenfrequency;

$D = \dfrac{b}{b_{cr}}$ – the damping ratio;

c – the elasticity coefficient;

b – the damping coefficient;

$b_{cr} = 2\sqrt{cm} = 2m\omega$ – the critical value of the damping coefficient;

b) Deformation of the connections

The most usual layouts of the components on the board [4], considering the direction of the perturbation force, are presented in Figure 2.

In order to determine the stresses acting upon the components, the well known methods of the strength of materials will be used, considering the simplifying hypotheses of a uniform structure, made of an isotropic material, acted upon by a uniformly distributed supplemental force produced by the weights of the components.

It can be seen in Figure 2 that, if only the connections are deformed, the system is reduced to a statically undetermined frame, for which the deformations, forces and moments may be deduced according to formulae in reference [5].

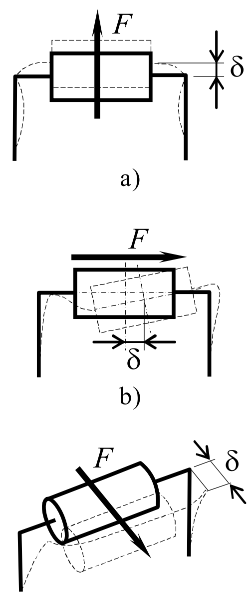

Fig. 2. The dynamic load on the electronic components

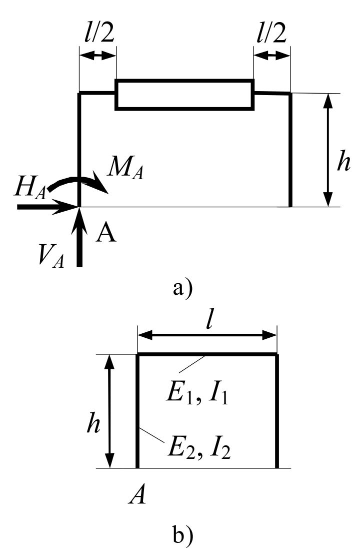

Fig. 3. The model for the case of connections deformation

The mechanical model is presented in

Figure 3, while the results are:

– for Fig. 2 a,

$$V_A = \frac{3Fhc}{l(1+6c)}, \tag{4}$$

$$H_A = \frac{F}{2}, \tag{5}$$

$$M_A = \frac{Fh}{4}\left(1+\frac{1}{6c+1}\right), \tag{6}$$

$$\delta = \frac{Fh^3}{24E_2I_2}\left(1+\frac{3}{6c+1}\right); \tag{7}$$

– for Fig. 2 b,

$$V_A = \frac{F}{2}, \tag{8}$$

$$H_A = \frac{3Fl}{2h(8+4c)}, \tag{9}$$

$$M_A = \frac{Fl}{8c+16}, \tag{10}$$

$$\delta = \frac{Fl^3}{48E_1I_1}\left(1-\frac{3}{2c+4}\right); \tag{11}$$

– for Fig. 2 c,

$$V_A = \frac{F^2h^2}{8E_2I_2}, \tag{12}$$

$$H_A = \frac{F}{2}, \tag{13}$$

$$M_A = \frac{Fh}{2}, \tag{14}$$

$$\delta = \frac{Fl^3}{48E_1I_1} - \frac{M_{tB}}{8E_1I_1} + \frac{Fh^3}{6E_2I_2}. \tag{15}$$

A particular case is the statically undetermined system (Fig. 4), with negligible damping [3].

The equivalent mechanical model is shown in Figure 4 e. In order to calculate the natural circular eigenfrequency, it is necessary to find the equivalent elasticity constant c^*.

Due to the symmetry of the mechanical system, only its left side will be considered. For this part,

$$c_1 = \frac{F}{\delta}, \tag{16}$$

where F is the force acting in point A and producing the displacement δ. For the model in Figure 4 c, displacements δ and φ are

$$\delta = \frac{Fh^3}{3EI} - \frac{M_Ah^2}{2EI}, \quad \varphi = \frac{Fh^2}{2EI} - \frac{M_Ah}{EI}, \tag{17}$$

where the unknown value is the moment M_A in the point A.

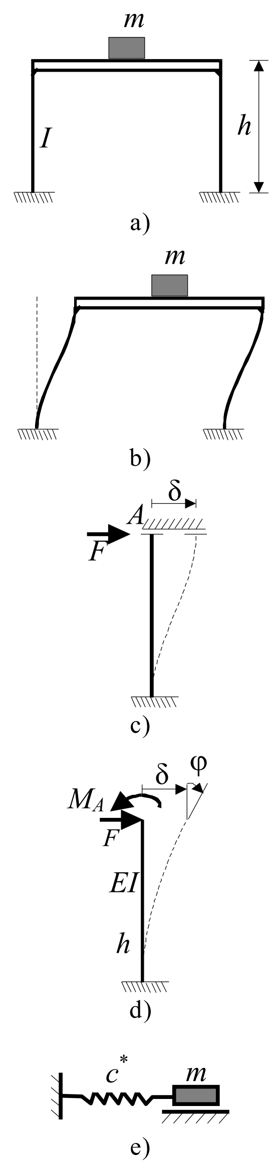

Fig. 4. The case of a statically undetermined system with negligible damping

Considering the necessary condition φ=0, the following results are obtained:

$$M_A = \frac{Fh}{2}\,, \tag{18}$$

$$\delta = \frac{Fh^3}{3EI} - \frac{Fh^3}{4EI} = \frac{Fh^3}{12EI}\,, \tag{19}$$

$$c_1 = \frac{F}{\delta} = \frac{12EI}{h^3}\,. \tag{20}$$

The equivalent elastic constant of the system can be calculated,

$$c^* = 2c_1 \tag{21}$$

while the natural circular eigenfrequency of the system is:

$$\omega = \sqrt{\frac{c^*}{m}} = \sqrt{\frac{24EI}{mh^3}}\,. \tag{22}$$

c) Stresses in the elastic and plastic domain

The analysis is made on the mechanical model in Figure 5, consisting of a beam *OA* upon which the force $F(t) = F_0 \cos \Omega t$ is acting in the point *A* [2].

The differential equation of the free vibrations of the beam is considered:

$$\frac{\partial^4 y}{\partial x^4} + \frac{\rho A}{EI}\frac{\partial^2 y}{\partial t^2} = 0\,. \tag{23}$$

For this equation, there will be searched solutions of the form

$$y(x,t) = Y(x)\cos(\Omega t)\,. \tag{24}$$

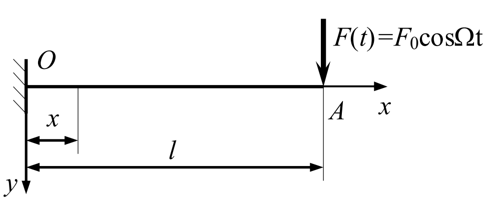

Fig. 5. The mechanical model for elasto-plastic deformations

By introducing this solution, differential equation (23) leads to

$$Y^{IV} - \frac{\rho A \Omega^2}{EI} Y = 0\,, \tag{25}$$

where the following notations have been used:
E – the elasticity module;
ρ – the density of the material;
A –cross-sectional area of the beam;
I – the geometric moment of inertia.

Proceeding according to the methods in the literature [2], the expression

$$Y(l) = \frac{F_0}{EI\alpha^3} \cdot \frac{\operatorname{ch}\alpha l \cdot \sin\alpha l - \operatorname{sh}\alpha l \cdot \cos\alpha l}{1 + \operatorname{ch}\alpha l \cdot \cos\alpha l}\,. \tag{26}$$

is obtained.

For the considered model [1], the natural mode of vibration shown in Figure 6 is obtained.

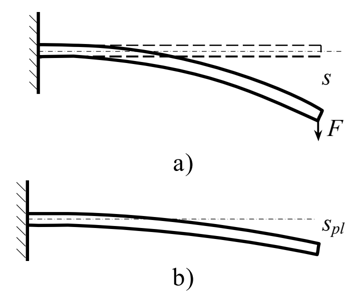

Fig. 6. The elastic and the plastic deformations of the beam

As can be seen, an increasing force *F*(*t*) will produce for the beginning elastic deformations, proportionally with the stress. If the force increases, the deformations may become plastic, while the function describing the deformation becomes nonlinear (Fig. 6 a). The maximum deformation *s* corresponds to the maximum value of the force *F*. When the force decreases to zero, the deformations disappear if the plastic limit was not exceeded. Otherwise, a plastic deformation of the beam will be visible (Fig. 6 b).

Experimental measurements were performed with simple oscillating systems made of copper wire, with the diameter *d*, the length *h* and the concentrated mass *m*. Using a regression model, a formula for the resonance transmissibility τ (the quality factor) [4] was determined,

$$\tau = \tau_{ref}\left(\frac{d}{d_{ref}}\right)^{\frac{3n-u-4}{n-1}}\left(\frac{m}{m_{ref}}\right)^{-\frac{n-2}{n-1}} \times \left(\frac{h}{h_{ref}}\right)^{-\frac{n-3}{n-1}}\left(\frac{a_0}{a_{ref}}\right)^{-\frac{n-2}{n-1}}, \tag{27}$$

where:

$$n=5.88,$$

$$u=3.9,$$

$$d_{ref}=1.36\cdot10^{-3}\text{m},$$

$$h_{ref}=1.5\cdot10^{-2}\text{m},$$

$$m_{ref}=7.5\cdot10^{-3}\text{kg},$$

$$a_{ref}=32\text{m/s}^2,$$

$$\tau_{ref}=13.$$

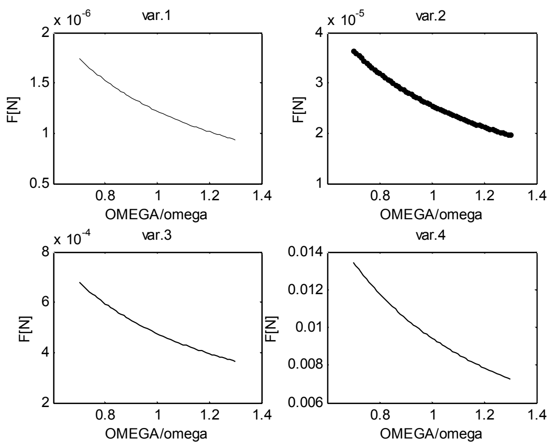

Fig. 7. The dynamic response curves

Dynamic response curves with respect to the relative circular frequency Ω/ω are shown in Figure 7, determined for m=7.5g, h=15.5mm, d=1.36mm, in four computation variants:

1) $\frac{F_0}{m}=1\text{m/s}^2$, $\tau=20$;

2) $\frac{F_0}{m}=3.2\text{m/s}^2$, $\tau=77$;

3) $\frac{F_0}{m}=10\text{m/s}^2$, $\tau=32$;

4) $\frac{F_0}{m}=32\text{m/s}^2$, $\tau=13$.

If an electronic component with two connectors is considered, the value of m in relation (27) is one half of the real mass of the element [4].

Laboratory experiments lead to the results shown in Figures 8-10.

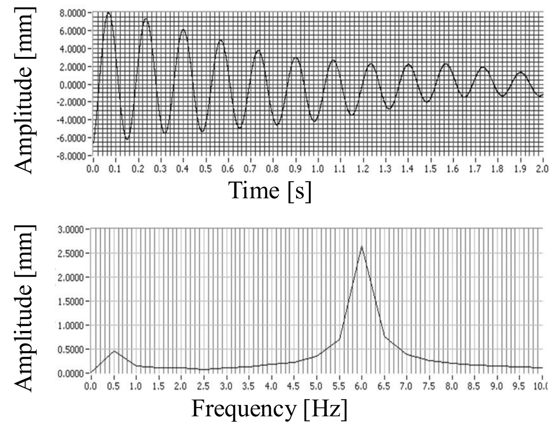

Fig. 8. Displacement

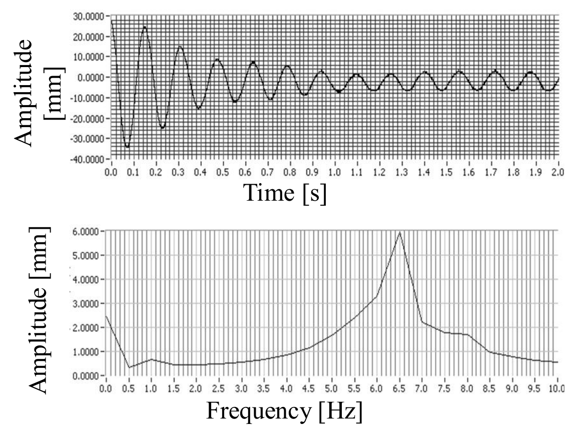

Fig. 9. Speed

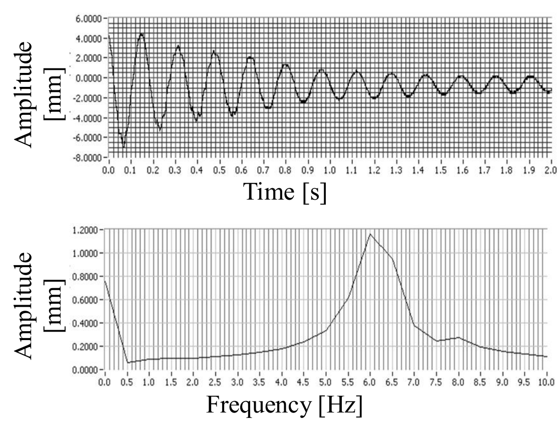

Fig. 10. Acceleration

d) The dynamic behavior of a cantilever layout

If the circuit board is vertical and the electronic component is horizontally placed (Fig. 11 a), due to the two connections, a rotation tendency around the central axis may occur [4].

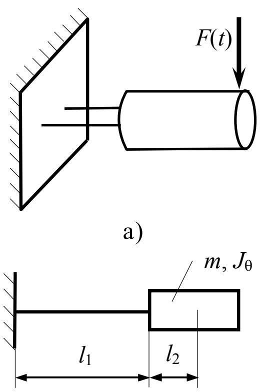

Fig. 11. Console setting and mechanical model

In order to consider also the rotation possibility, the mechanical model in Figure 5 must be transformed as shown in Figure 11 b. In this situation, two natural frequencies, one for each type of motion (translation and rotation), will appear [5]. The corresponding natural vibration forms are shown in Figure 12 a and b, respectively.

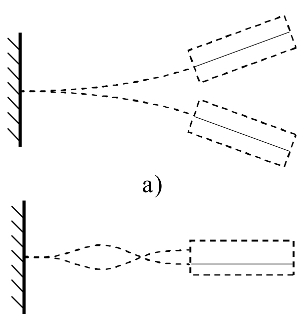

Fig. 12. The natural forms of vibration for translation and rotation

For the translation vibration, condition l_2=0 must be applied, not necessary in the case of the rotation. The natural circular eigenfrequencies are

$$\omega_{trans} = \sqrt{\frac{c}{m}}, \quad \omega_{rot} = \sqrt{\frac{c_\varphi}{m_\varphi}}, \tag{28}$$

where

$$c = \frac{3EI}{l_1^3}, \quad c_\varphi = \frac{3EI}{l_1}, \tag{29}$$

while I is the geometric moment of inertia of the connection section.

3. CONCLUSIONS

The study, made for some important practical situations, leads to the conclusion that the effects of the dynamic stresses induced in an electronic circuit board, by mechanical vibrations, must be taken into account, due to their influence upon the reliability and functionality of the system. These two issues have major implications in the cost of manufacturing and operation of electronic devices. The importance of the study is accounted for by the the large scale usage in present times of such electronic circuit elements.

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