# MODEL REFERENCE ADAPTIVE CONTROL OF A VECTOR CONTROLLED THREE PHASE INDUCTION MACHINE

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Abstract: Model reference adaptive control (MRAC) of three-phase induction machine in rotor field coordinated is proposed in this paper. The proposed adaptive structure is in direct form, such that the controller parameters are obtained on-line. The adaptive mechanism of the parameters is obtained by additive composing of two terms: the first will support a gradient adjustment law and the second will comport an adjustment that includes a sigmoid function specific for variable structure control. The last component improves the transient and steady state response by eliminating the small oscillations of the closed loop response around the equilibrium state in order to obtain a zero tracking error. The adaptive mechanism assures the robustness to the external disturbances and to the unmodeled dynamics. Matlab/Simulink based simulation results will show the effectiveness of the proposed solution.

Keywords: rotor field oriented, model reference adaptive control, induction motor, Matlab/Simulink.

## 1. INTRODUCTION

High performance electric drive systems require a very accurate knowledge of the electrical machine parameters. From parameter variation point of view, due to the inflexible conventional methods, i.e. no load and locked rotor tests (*IEEE Std, 2004*), the on-line parameter identification methods are more advantageous. The recursive least square identification algorithm is the most used method (Cirrincione, *et al.,* 2003; Netto, *et al.,* 2004). Anyway, in order to obtain the initial conditions of the parameters, the classical tests must be done. In order to avoid the above mentioned disadvantage, a modern technique based on MRAC (Filipescu, 1994; Ioannou, and Fidan 2006) of a vector controlled three-phase induction machine is proposed in this paper. The reference model response is compared with the speed of the three-phase induction motor, and the tracking error is used in order to adjust the control parameters (Cheng-Hung Tsai, and Yeh, 2009; Azzolin, and Gründling, 2009; Li, 2011).

The MRAC system guarantees the asymptotic cancelation of the tracking error, therefore obtaining the perfect matching of the three-phase IM response with the reference model. The paper is structured as follows: the mathematical model of the three-phase induction machine in rotor field oriented reference frame is presented in Section 2, the MRAC in direct form is widely described in Sections 3 and 4, and the corresponding simulation results are provided in Section 5.

## 2. MATHEMATICAL MODEL OF THE VECTOR CONTROLLED THREE-PHASE INDUCTION MOTOR

The mathematical model of the induction motor driven in rotor field coordinates frame, with the assumption of maintaining rotor magnetizing current at constant value (Gaiceanu, *et al.* 2000) is presented in standard form of the state space:

$$(1)\quad \begin{bmatrix} \overset{\circ}{\omega}_m(t) \\ \overset{\circ}{q}(t) \end{bmatrix} = \begin{bmatrix} -\dfrac{F_v}{J} & 0 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \omega_m(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} \dfrac{K_m}{J} \\ K_q \end{bmatrix} \cdot i_{sq}(t) + \begin{bmatrix} -\dfrac{1}{J} \\ 0 \end{bmatrix} \cdot T_l(t)$$

with specific constants:

$$(2)\quad K_q = \frac{1}{\tau_R i_{mR}},\; K_m = \frac{2}{3} p \frac{M}{1+\sigma_R} i_{mR}$$

in which: $i_{mR} = i_{sd}$ - the magnetizing current and the flux component current, $i_{sq}$- the torque component current, $\omega_m$ - the instantaneous mechanical angular velocity of the rotor, $T_l$ - load torque of the induction motor, *q*- the angular position of the rotor field, *J*- the combined inertia of the motor and load, $F_v$- the viscous friction coefficient, *M*- mutual inductance between the stator and rotor *d*, *q* equivalent windings, $\tau_R$ - the rotor time constant, $\sigma_R$ - the rotor leakage factor, *p*- the number of pole pairs, $K_m$- torque constant.

## 3. MODEL REFERENCE ADAPTIVE CONTROL

The transfer function of the dynamical system (1) is

$$(3)\quad H_p(s) = K_p \frac{N_p(s)}{D_p(s)}$$

in which:

- the degree of the monic polynomials $N_p(s)$ and $D_p(s)$ are $m_p$, and $n_p$ respectively;
- the sign of the gain $K_p$ is known;
- the relative degree $n_p^* = n_p - m_p$ of $H_p(s)$ is known.

The transfer function of the reference model is given by

$$(4)\quad H_m(s) = K_m \frac{N_m(s)}{D_m(s)};$$

in which:

- the degree of the monic Hurwitz polynomials $N_m(s)$ and $D_m(s)$ are $p_m$ and $n_m$, respectively;
- the relative degree $n_m^* = n_p^*$ in order to obtain a zero tracking error.

The model reference adaptive control synthesis with unitary relative degree is presented as follows.

## 4. DESIGN OF THE ADAPTIVE CONTROLLER

### *4.1. The gradient adjustment law*

The filtered signals vector is

$$(5)\quad \mathbf{v}(t) = \left[\mathbf{v}_{\mathbf{u}}^T(t) \quad \mathbf{v}_{\mathbf{y}_p}^T(t) \quad y_p(t) \quad r(t)\right]^T \in \Re^{2n_p}$$

in which *r*(t) is the reference signal, $y_p$ (t) is the output signal of the process, the filters dynamic connected at the control $\mathbf{v}_{\mathbf{u}} \in \Re^{n_p - 1}$, and at the output of the plant $\mathbf{v}_{\mathbf{y}} \in \Re^{n_p - 1}$ are described below (Filipescu, A., 1994)

$$(6)\quad \begin{cases} \overset{\circ}{\mathbf{v}}_{\mathbf{u}} = \Lambda \mathbf{v}_{\mathbf{u}} + \mathbf{h}u \\ \overset{\circ}{\mathbf{v}}_{\mathbf{y}} = \Lambda \mathbf{v}_{\mathbf{y}} + \mathbf{h}y_p \end{cases}$$

The pair ($\Lambda$, $\mathbf{h}$) can be chosen in controllable canonical form: the matrix $\mathbf{\Lambda} \in \Re^{(n_p-1)\times(n_p-1)}$ and the vector $\mathbf{h} \in \Re^{n_p}$ (Filipescu, A., 1994),

$$(7)\quad \begin{aligned} \mathbf{v}_{\mathbf{u}}(s) &= (s\mathbf{I} - \mathbf{\Lambda})^{-1} \mathbf{h} U(s) \\ \mathbf{v}_y(s) &= (s\mathbf{I} - \mathbf{\Lambda})^{-1} \mathbf{h} Y_p(s) \end{aligned}$$

such that (Filipescu, A., 1994):

$$(8)\quad \det(s\mathbf{I} - \mathbf{\Lambda}) = N_m(s)$$

For a gradient adjustment law the following parameters are obtained:

$$(9)\quad \begin{cases} \overset{\circ}{\theta}_{gu} = -\gamma_g \cdot sign(k_p) \cdot \mathbf{v}_u \cdot e_0 \\ \overset{\circ}{\theta}_{gy} = -\gamma_g \cdot sign(k_p) \cdot \mathbf{v}_y \cdot e_0 \\ \overset{\circ}{\theta}_{gp} = -\gamma_g \cdot sign(k_p) \cdot y_p \cdot e_0 \\ \overset{\circ}{\theta}_{gr} = -\gamma_g \cdot sign(k_p) \cdot r \cdot e_0 \end{cases}$$

which guarantee the asymptotic ancellation of the tracking error $\lim_{t\to\infty} e_0(t) = 0$, $\gamma_g$ being a positive constant.

The controller parameter vector

$$(10)\quad \theta_g \in \Re^{2n_p} = \left[\theta_{gu}^T(t) \quad \theta_{gy_p}^T(t) \quad \theta_{gp}(t) \quad \theta_{gr}(t)\right]^T$$

includes the subvectors $\theta_{gu} \in \Re^{n_p-1}$ and $\theta_{gy_p} \in \Re^{n_p-1}$.

The corresponding adaptive law is considered by a scalar product of the vectors $\theta_g$ and $\mathbf{v}$

$$(11)\quad u(t) = \theta_g^T \mathbf{v}(t);$$

*4.2. Variable structure adaptive law*

Variable structure adaptive control with parameter adjustment based on signum function leads to a fast and smooth system response, but with asymptotically small oscillations around the equilibrium point. Parameter adjustment law will include an approximation of *sign(x)* function by using a *k*-sigmoid function (Filipescu, 1994; Ioannou et al., 2006):

$$(12)\quad \begin{aligned} \theta_{vu} &= \bar{\theta}_{vu} sign(k_p) sign(e_0 \mathbf{v}_u) \cong \bar{\theta}_{vu} \frac{e^{ke_0\mathbf{v}_u} - 1}{e^{ke_0\mathbf{v}_u} + 1} sign(k_p) \\ \theta_{vy_p} &= \bar{\theta}_{vy_p} sign(k_p) sign(e_0 \mathbf{v}_{y_p}) \cong \bar{\theta}_{vy_p} \frac{e^{ke_0\mathbf{v}_{y_p}} - 1}{e^{ke_0\mathbf{v}_{y_p}} + 1} sign(k_p) \\ \theta_{y_p} &= \bar{\theta}_{y_p} sign(k_p) sign(e_0 y_p) \cong \bar{\theta}_{y_p} \frac{e^{ke_0 y_p} - 1}{e^{ke_0 y_p} + 1} sign(k_p) \\ \theta_r &= \bar{\theta}_r sign(k_p) sign(e_0 r) \cong \bar{\theta}_r \frac{e^{ke_0 r} - 1}{e^{ke_0 r} + 1} sign(k_p) \end{aligned}$$

$$(13)\quad \begin{aligned} \overset{o}{\bar{\theta}}_{vu} &= -\boldsymbol{\lambda}_{\mathbf{vu}} \bar{\theta}_{vu} - \gamma_v \left| e_0 \mathbf{v}_u \right| \\ \overset{o}{\bar{\theta}}_{vy_p} &= -\boldsymbol{\lambda}_{\mathbf{vy}_p} \bar{\theta}_{vy_p} - \gamma_v \left| e_0 \mathbf{v}_{y_p} \right| \\ \overset{o}{\bar{\theta}}_{y_p} &= -\lambda_{y_p} \bar{\theta}_{y_p} - \gamma_v \left| e_0 y_p \right| \\ \overset{o}{\bar{\theta}}_r &= -\lambda_r \bar{\theta}_r - \gamma_v \left| e_0 r \right| \end{aligned}$$

in which $\boldsymbol{\lambda}_{\mathbf{vu}}, \boldsymbol{\lambda}_{\mathbf{vy}_p} \in \Re^{(n_p-1)\times(n_p-1)}$ are positive diagonal matrices, $\lambda_{y_p}$, $\lambda_r$, $\gamma_v$ are positive constants.

The variable structure adaptive control is given by

$$(14)\quad \begin{aligned} u(t) &= \theta_{\mathbf{v}}^T \mathbf{v}(t); \quad \theta_{\mathbf{v}}(t) \\ &= \left[ \theta_{\mathbf{vu}}^T(t) \quad \theta_{\mathbf{vy}_p}^T(t) \quad \theta_{y_p}(t) \quad \theta_r(t) \right]^T \end{aligned}$$

*4.3. Compound law adaptive control*

The control law parameters vector is obtained by adding two terms (Fig 2):

$$(15)\quad \theta = \theta_g(t) + \theta_v(t), \quad u(t) = \theta^T(t)\mathbf{v}(t)$$

$\theta_g \in \Re^{2n}$ represent parameters vector with gradient adjustment law, and $\theta_v \in \Re^{2n}$ is parameters vector of variable structure.

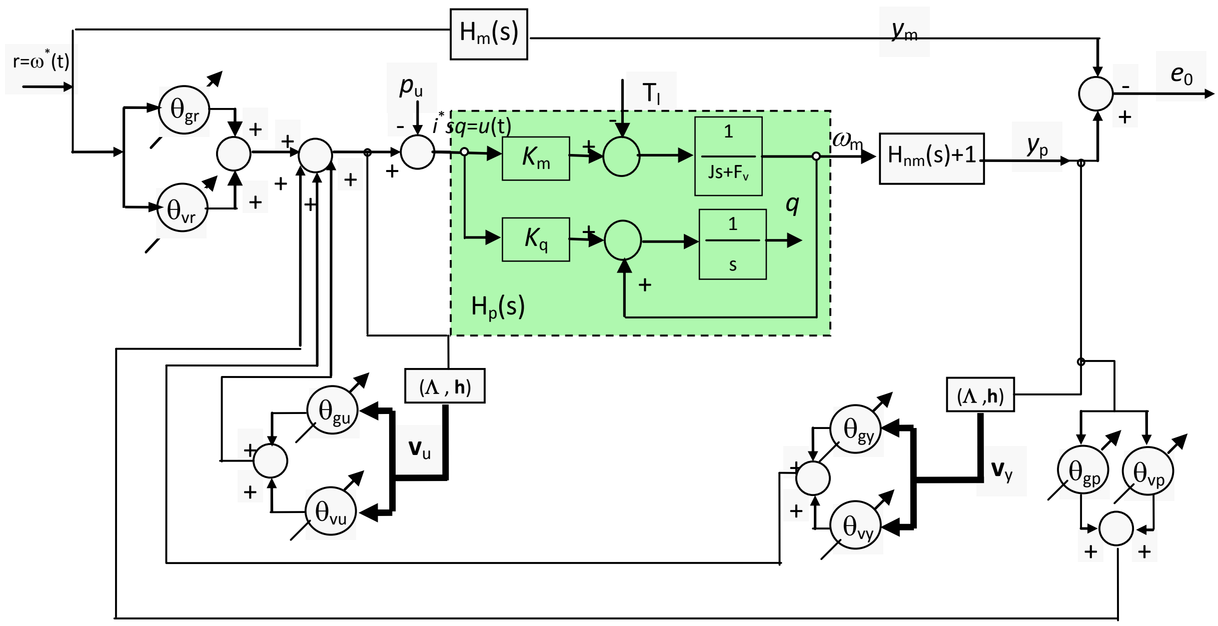


Fig.1. MRAC with vector controlled drive system for three-phase IM

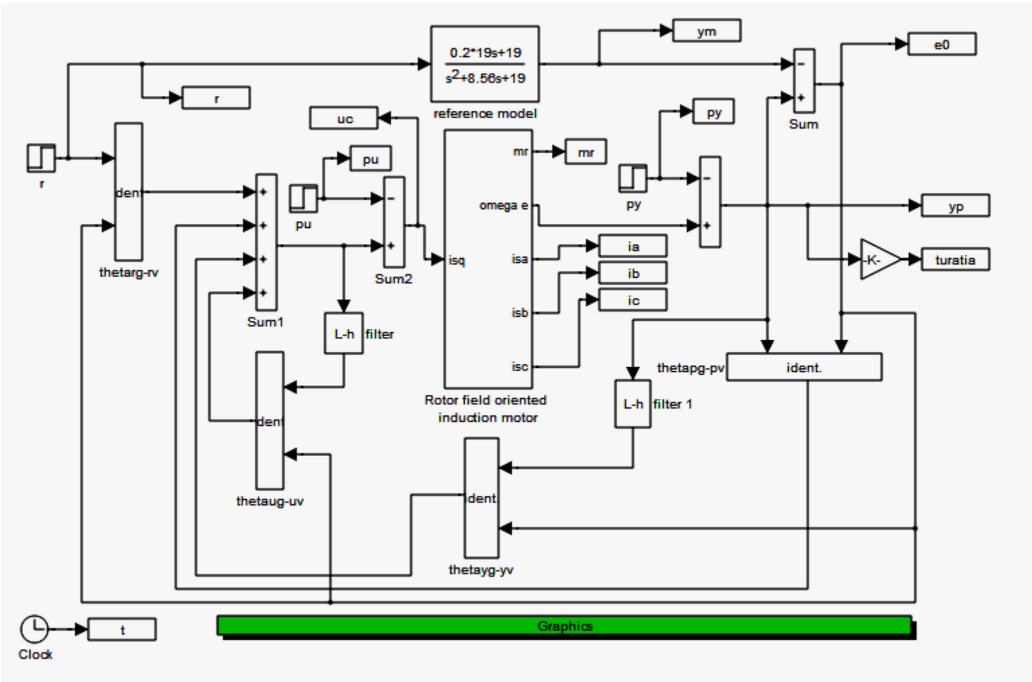


Fig.2. Matlab/Simulink implementation of the MRAC with vector controlled drive system for three-phase IM

## 5. SIMULATION RESULTS

The MRAC of three-phase induction machine (IM) is shown in Fig. 1, in which $H_m$ (s) is transfer function of the reference model and $H_{nm}(s)$ is the transfer function of the unmodelled dynamics. The Matlab/Simulink implementation is depicted in Fig.2. The Matlab/Simulink-based compound law with both components gradient adjustment and variable structure adaptive control is presented in Fig.3.

The mathematical model of a 7.5 [kW], 1480 [rpm] induction motor under a load torque of 2,38 [Nm] with a constant magnetizing current $i_{mR}=i_{sd}=1.8$ [A] is shown in the Figure 4. By using adaptive control (14), the angular velocity $\omega_m$ and the rotor field magnetizing angle (*q*) are delivered.

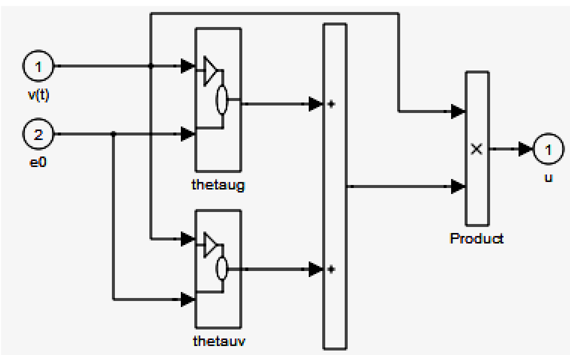


Fig.3 The compound law with both components: gradient adjustment and variable structure adaptive control.

The Forward Vector Transformation (FVT) block (Fig.4) transform the two phase (rotor field reference frame) components into three phase input currents (stator reference frame) by using Park and Clarke transformations. In order to perform these transformations, the instantaneous angle of the rotor field, *q*, is required.

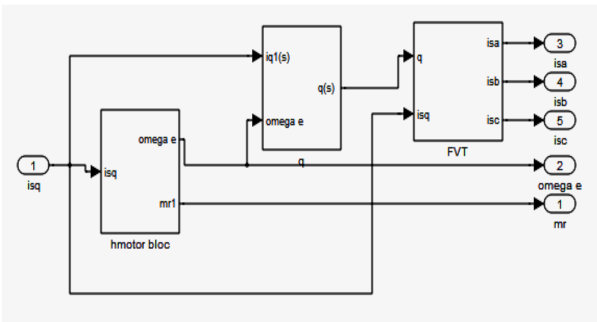


Fig.4 Matlab/Simulink implementation of the three-phase IM in rotor field reference frame

The simulation results of the adaptive electric drive system for a starting with no load, followed at $t$ = 0.5[s] of an applied load torque to three-phase induction motor shaft, are shown in Fig. 5 and Fig. 6.

This control law (14) assures stability and regulating properties (confirming the robustness character of the law), and makes smooth transient response and zero tracking error; the magnitude of oscillations is related by the $\gamma_g$ parameter from the adaptive control by gradient adjustment law. Thus, the asymptotic performances will be assured by gradient adjustment component.

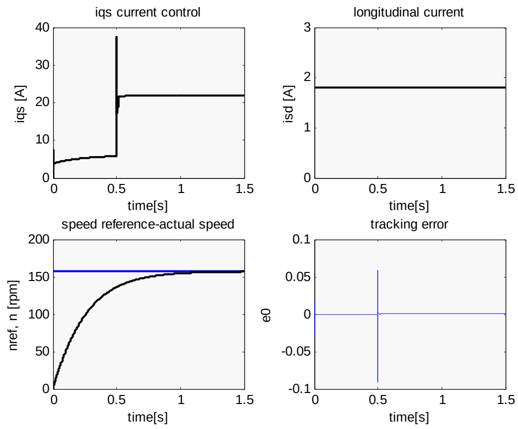


Fig.5. The main simulation results: the torque current component ($i_{sq}$), the flux component ($i_{sd}$), the speed reference ($n_{ref}$) and the actual one ($n$), the tracking error ($e_0$).

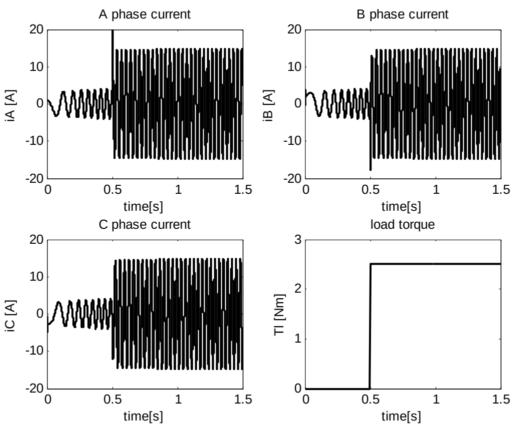


Fig. 6. The phase currents under the load torque

By using $k$-sigmoid function the small oscillations of the closed loop response around the equilibrium state are eliminated. To obtain a fast convergence rate, a high value of $\gamma_v$ parameter has been chosen.

## 6. CONCLUSIONS

In order to evaluate the proposed MRAC three-phase IM, the simulation results based Matlab/Simulink software are provided. The model reference adaptive control, in direct form, unnormalized of three-phase induction machine has been presented. A robust electric drive system is obtained, the actual speed being insensible to load variations (Fig.5).

The adaptive control used in this paper exploit the full advantages of both components: the gradient adjustment law assures the asymptotic performances, and the variable structure control (based on adjustment that includes a sigmoid function) assures smooth response around the equilibrium point, robustness of the system, and a zero tracking error.

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