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SPATIO-TEMPORAL DYNAMICS IN CELLULAR NEURAL NETWORKS

Liviu Goraş

Faculty of Electronics, Telecommunications and Information Technology, "Gh. Asachi" Technical University of Iaşi and Institute for Computer Science, Romanian Academy, Iasi Branch
e-mail: lgoras@etc.tuiasi.ro

Abstract: Analog Parallel Architectures like Cellular Neural Networks (CNN's) have been thoroughly studied not only for their potential in high-speed image processing applications but also for their rich and exciting spatio-temporal dynamics. An interesting behavior such architectures can exhibit is spatio-temporal filtering and pattern formation, aspects that will be discussed in this work for a general structure consisting of linear cells locally and homogeneously connected within a specified neighborhood. The results are generalizations of those regarding Turing pattern formation in CNN's. Using linear cells (or piecewise linear cells working in the central linear part of their characteristic) allows the use of the decoupling technique – a powerful technique that gives significant insight into the dynamics of the CNN. The roles of the cell structure as well as that of the connection template are discussed and models for the spatial modes dynamics are made as well.

Keywords: analog parallel architectures, Cellular Neural Networks, spatio-temporal dynamics, spatial filters

1. INTRODUCTION[1]

Spatio-temporal dynamics leading to pattern formation have been thoroughly studied in various fields including autonomous cellular neural networks (CNN's) (Turing, 1952; Chua, and Yang, 1988 a,b; Murray, 1993; Roska, and Vanderwalle, 1993; Crounse and Chua, 1995a; Chua, *et al.,* 1995; Crounse and Chua, 1995b; Roska and Chua, 1995; Roska, *et al.,* 1995; Zarandy and Roska , 1997; Shi, 1998; Bing, *et al.,* 1998; Frasca, *et al.,* 2004; Arena, *et al.,* 2004). Among various types of patterns, Turing ones have been studied in connection to an architecture consisting of two-port second order cells sandwiched between two resistive grids (Goras, *et al.,* 1995; Goras and Chua, 1995; Goras, 2002; Goras and Chua, 1996; Teodorescu, and Goras, 1997; Goras and Chua, 1997a,b; David, *et al.*, 2004; Ungureanu, *et al.,* 2006). The specific feature of Turing patterns is that the isolated cells are stable while the dynamics of the array can exhibit unstable spatial modes. If the cells are piecewise linear, a powerful method of investigation is the decoupling technique basically consisting of a change of variable chosen according to the boundary conditions. The transformed differential equations corresponding to each spatial mode are decoupled, parts of them having unstable solutions – a necessary condition for pattern formation. The competition of the unstable spatial modes leads to a pattern which depends on the shape of the dispersion curve, initial conditions and on the nonlinearity of the cells characteristics. Of course, the method is valid only for the central linear part of the cell characteristics but offers useful insight on the shape of final pattern obtained after the nonlinearity has been reached (Goras and Chua, 1995).

Basically, two mechanisms were used so far to limit the pattern evolution: either the nonlinearity of the

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cell characteristics or “freezing” the transient typically before any nonlinearity has been reached. If the emerging pattern is frozen before the signals leave the central linear part of the cell characteristics, the CNN behaves as a spatial time variable filter, the spatial frequency response being dependent on the moment the transient has been stopped (Shi, 1998; Goras, 2002). Various aspects regarding the spatio-temporal dynamics of two-grid coupled CNN’s and their applications as texture classification have been reported in (Goras and Chua, 1996; Teodorescu, and Goras, 1997; Goras and Chua, 1997a,b; David, *et al.*, 2004; Ungureanu, *et al.,* 2006; Goras, *et al.,* 2007; Alecsandrescu and Goras, 2008).

The two-grid coupled CNN is a rather special case of homogeneous parallel architecture, derived from the reaction-diffusion model proposed by Turing. The basic idea of having arrays with one or perhaps several bands of unstable spatial modes can be implemented using a more flexible architecture. Even though from a theoretical point of view architectures with any degree of complexity can be imagined, implementation constraints make large structures with cell order and/or neighboring radii higher than two being rather unrealistic.

2. THE BASIC ARCHITECTURE

For the sake of simplicity, in the following we consider a 1D array with the architecture shown in Fig.1. Generalizations to two dimensions are straightforward. The array consists of linear (or piecewise linear, working in the central linear part) cells represented by admittances denoted by Y(s) and coupled between them as well as with the inputs, using voltage controlled current sources over a neighborhood of radius r, Nr. The template elements have been denoted by Ak for inter-cell connections and by Bk for the sources connections. The neighborhood dimension has been chosen the same for both cases, a fact that does not restrict the generality as any of the template coefficients might be zero.

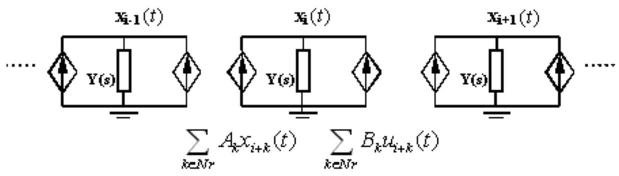

Fig.1. 1D array architecture.

Considering $s \leftrightarrow d/dt$, *Y(s)* is a linear integro-differential operator of the form:

$$(1)\quad Y(s) = \frac{Q(s)}{P(s)} = \frac{\sum_{l=0}^{q} q_l s^l}{\sum_{n=0}^{p} p_n s^n}$$

in particular a real-positive function where $Q(s)$ and $P(s)$ are polynomials in the variable s.

In this case, the equations that formally describe the network are:

$$(2)\quad Y(s)x(t) = \sum_{k\in Nr} A_k x_{i+k}(t) + \sum_{k\in Nr} B_k u_{i+k}(t)$$

where $s \leftrightarrow d/dt$ or

$$(3)\quad \begin{aligned}\sum_{l=0}^{q} q_l \frac{d^l}{dt^l} x_i(t) &= \sum_{k\in Nr}\sum_{n=0}^{p} A_k p_n \frac{d^n}{dt^n} x_{i+k}(t) + \\ &+ \sum_{k\in Nr}\sum_{n=0}^{p} B_k p_n \frac{d^n}{dt^n} u_{i+k}(t), \quad i = 0,1,\ldots,M-1\end{aligned}$$

Obviously, the above relations represent a set of coupled integro-differential equations which can be elegantly solved by the decoupling technique.

3. THE DECOUPLING TECHNIQUE

In the following we will use the decoupling technique (Goras and Chua, 1995) to study the spatio-temporal behavior of the network.
We consider the change of variables

$$(4)\quad x_i(t) = \sum_{m=0}^{M-1} \Phi_M(i,m)\hat{x}_m(t)$$

$$(5)\quad u_i(t) = \sum_{m=0}^{M-1} \Phi_M(i,m)\hat{u}_m(t)$$

where the M functions $\Phi_M(i,m)$ are orthogonal with respect to the scalar product in C^M, i.e.,

$$(6)\quad \begin{aligned}<\Phi_M(i,m),\Phi_M(i,n)> &= \sum_{i=0}^{M-1} \Phi_M^*(i,m)\Phi_M(i,n) = \\ &= \sum_{i=0}^{M-1} \Phi_M(m,i)\Phi_M(i,n) = \delta_{mn}\end{aligned}$$

so that $\hat{x}_m$ and $\hat{u}_m$ can be expressed, by means of the inversion formulas:

$$(7)\quad \begin{aligned}\hat{x}_m(t) &= \sum_{i=0}^{M-1} \Phi_M^*(m,i)x_i(t) \\ \hat{u}_m(t) &= \sum_{i=0}^{M-1} \Phi_M^*(m,i)u_i(t)\end{aligned} \qquad m = 0,...,M-1$$

where

$$(8)\quad \Phi_M(m,i) = \Phi_M^*(i,m)$$

For ring boundary conditions $\Phi_M(m,i)$ have the form $\Phi_M(i,m) = e^{j2\pi mi/M} = e^{j\omega_m i} = e^{j\omega_0 mi}$ where $\omega_0=2\pi/M$ and $\omega_m=2\pi m/M=\omega_0 m$.

If $\Phi_M(i+k,m) = e^{j2\pi mk/M}\Phi_M(i,m)$ which happens in the case of $\Phi_M(i,m) = e^{j2\pi mi/M}$, the action of the spatial operators A and B on $\Phi_M(i,m)$ gives

$$(9)\quad \sum_{k\in Nr} A_k \Phi_M(i+k,m) = K_A(m)\Phi_M(i,m)$$

$$(10)\quad \sum_{k\in Nr} B_k \Phi_M(i+k,m) = K_B(m)\Phi_M(i,m)$$

where

$$(11)\quad \begin{aligned}K_A(m) &= \sum_{k=-r}^{r} A_k e^{j2\pi mk/M} = \\ & A_0 + \sum_{k=1}^{r}(A_k + A_{-k})\cos 2\pi mk/M + \\ & + j\sum_{k=1}^{r}(A_k - A_{-k})\sin 2\pi mk/M\end{aligned}$$

and

$$(12)\quad \begin{aligned}K_B(m) &= \sum_{k=-r}^{r} B_k e^{j2\pi mk/M} = \\ &= B_0 + \sum_{k=1}^{r}(B_k + B_{-k})\cos 2\pi mk/M + \\ & + j\sum_{k=1}^{r}(B_k - B_{-k})\sin 2\pi mk/M\end{aligned}$$

Thus $\Phi_M(i,m)$ are eigenfunctions of the spatial operators represented by the A and B templates and $K_A(m)$ and $K_B(m)$ are the corresponding spatial eigenvalues which are complex in general and depend on the parameters of the template and on the mode. For symmetric templates, $A_{-k}=A_k$, the spatial eigenvalues are real.

$$(13)\quad K_A(m) = A_0 + 2\sum_{k=1}^{r} A_k \cos\frac{2\pi mk}{M}$$

$$(14)\quad K_B(m) = B_0 + 2\sum_{k=1}^{r} B_k \cos\frac{2\pi mk}{M}$$

In particular, for symmetric templates with r=1

$$(15)\quad K_A(m) = A_0 + 2A_1\cos\omega(m) = A_0 + 2A_1\cos\frac{2\pi}{M}m$$

and for symmetric templates with r =2

$$(16)\quad \begin{aligned}K_A(m) &= A_0 + 2A_1\cos\omega(m) + 2A_2\cos 2\omega(m) = \\ &= A_0 + 2A_1\cos\frac{2\pi}{M}m + 2A_2\cos\frac{4\pi}{M}m\end{aligned}$$

with similar expressions for $K_B(m)$.

Using the above change of variable and the properties of $\Phi_M(i,m)$, equations (3) become successively

$$\begin{aligned}&\sum_{l=0}^{q} q_l \frac{d^l}{dt^l}\sum_{m=0}^{M-1}\Phi_M(m,i)\hat{x}_m(t) = \sum_{k\in Nr}\sum_{n=0}^{p} A_k p_n \frac{d^n}{dt^n}\sum_{m=0}^{M-1}\Phi_M(m,i+k)\hat{x}_m(t) + \\ &+ \sum_{k\in Nr}\sum_{n=0}^{p} B_k p_n \frac{d^n}{dt^n}\sum_{m=0}^{M-1}\Phi_M(m,i+k)\hat{u}_m(t),\ i=0,1,...,M-1\end{aligned}$$

$$\begin{aligned}&\sum_{m=0}^{M-1}\Phi_M(m,i)\sum_{l=0}^{q} q_l \frac{d^l}{dt^l}\hat{x}_m(t) = \sum_{m=0}^{M-1}\sum_{k\in Nr} A_k\Phi_M(m,i+k)\sum_{n=0}^{p} p_n\frac{d^n}{dt^n}\hat{x}_m(t) + \\ &+ \sum_{m=0}^{M-1}\sum_{k\in Nr} B_k\Phi_M(m,i+k)\sum_{n=0}^{p} p_n\frac{d^n}{dt^n}\hat{u}_m(t),\quad i=0,1,...,M-1\end{aligned}$$

$$\begin{aligned}&\sum_{m=0}^{M-1}\Phi_M(m,i)\sum_{l=0}^{q} q_l \frac{d^l}{dt^l}\hat{x}_m(t) = \sum_{m=0}^{M-1} K_A(m)\Phi_M(m,i)\sum_{n=0}^{p} p_n\frac{d^n}{dt^n}\hat{x}_m(t) + \\ &+ \sum_{m=0}^{M-1} K_B(m)\Phi_M(m,i)\sum_{n=0}^{p} p_n\frac{d^n}{dt^n}\hat{u}_m(t),\quad i=0,1,...,M-1\end{aligned}$$

If we take the scalar product of both sides of the last equations with $\Phi_M(n,i)$ and then replace the index n with m, we have

$$(17)\quad \begin{aligned}&\sum_{l=0}^{q} q_l \frac{d^l}{dt^l}\hat{x}_m(t) = K_A(m)\sum_{n=0}^{p} p_n\frac{d^n}{dt^n}\hat{x}_m(t) + \\ &+ K_B(m)\sum_{n=0}^{p} p_n\frac{d^n}{dt^n}\hat{u}_m(t),\ m=0,1,...,M-1\end{aligned}$$

which can be written symbolically either

$$(18)\quad Y(s)\hat{x}_m(t) = K_A(m)\hat{x}_m(t) + K_B(m)\hat{u}_m(t)$$

or

$$(19)\quad Q(s)\hat{x}_m(t) = K_A(m)P(s)\hat{x}_m(t) + K_B(m)P(s)\hat{u}_m(t)$$

and corresponds to the differential equation satisfied by the m-th spatial mode. Thus, the equations have been decoupled, the new variables being the amplitudes of the spatial modes of the cell signals.

4. TRANSFER FUNCTIONS *AND* DISPERSION CURVES

The transfer functions that can be associated to equations (18,19), valid for the magnitude of each spatial mode are

$$(20)\quad H_m(s)=\frac{\hat{x}_m(s)}{\hat{u}_m(s)}=\frac{K_B(m)}{Y(s)-K_A(m)}=\frac{K_B(m)Z(s)}{1-K_A(m)Z(s)}= \\ =\frac{K_B(m)P(s)/Q(s)}{1-K_A(m)P(s)/Q(s)},\ m=0,1,...M$$

where $\hat{x}_m(s)$ and $\hat{u}_m(s)$ are the Laplace transforms of $\hat{x}_m(t)$ and $\hat{u}_m(t)$ respectively.

The transfer functions correspond to a feedback system for each of the modes as depicted in the figure below where Z(s)=1/Y(s).

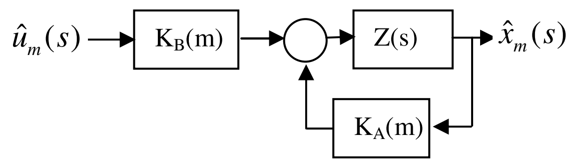

Fig.2. Feedback schematic for the spatial mode m

The characteristic polynomial of the m-th mode is

$$(21)\quad R(s)=Q(s)-K_A(m)P(s)$$

Thus, the stability and dynamics of the spatial modes will depend both on the A-template eigenvalues and on the cell admittance Y(s)=Q(s)/P(s). The spatio-temporal dynamics of the array can be studied using classical methods from feedback/control theory such as the root locus and Nyquist criterion and, of course, the Routh-Hurwitz test, all valid for each of the spatial modes, conveniently modified for non-symmetric templates.

For the particular case Y(s)=s, (i.e., Q(s)=s, P(s)=1) and $B_k=0$ we obtain the following set of decoupled autonomous first order differential equations:

$$(22)\quad \frac{d\hat{x}_m(t)}{dt}=K_A(m)\hat{x}_m(t)\qquad m=0,...,M-1$$

The roots of the characteristic equation are $s=K_A(m)$, so that the dispersion curve is a straight line with respect to K_A.

For Y(s)=s+1 we obtain a similar equation (the -1 constant can be always absorbed by A_0 in $K_A(m)$:

$$(23)\quad \frac{d\hat{x}_m(t)}{dt}=\left(-1+K_A(m)\right)\hat{x}_m(t)\qquad m=0,...,M-1$$

The roots of the m-th characteristic equation are thus $s_m=-1+K_A(m)$, so that, again, the dispersion curve is a straight line with respect to $K_A(m)$ but not with m.

$K_A(m)$ has the expression

$$(24)\quad K_A(m)=A_0+2A_1\cos\frac{2\pi}{M}m$$

for first order neighborhood and

$$(25)\quad K_A(m)=A_0+2A_1\cos\frac{2\pi}{M}m+2A_2\cos\frac{4\pi}{M}m= \\ =A_0-2A_2+2A_1\cos\frac{2\pi}{M}m+4A_2\cos^2\frac{2\pi}{M}m$$

for second order neighborhood. For this last case, the roots of $K_A(m)=0$ are:

$$(26)\quad m_1=\frac{M}{2\pi}\left(\arccos\left(\frac{-A_1+\sqrt{A_1^2-4A_0A_2+8A_2^2}}{4A_2}\right)\right)$$

$$(27)\quad m_2=\frac{M}{2\pi}\left(\pi-\arccos\left(\frac{A_1+\sqrt{A_1^2-4A_0A_2+8A_2^2}}{4A_2}\right)\right)$$

When real, the above roots represent the intersection of the dispersion curve with the abscissa axis in the hypothesis of a continuous variation of m.

Thus, if the quantity under the square root is positive and the module of the argument of the *arcos* function is less than unity, the array might have poles on the positive real axis of the complex plane on the condition that there is at least one integer m in the domain of unstable modes, i.e., for which $K_A(m)$ is positive. In such a case the unstable spatial modes will increase according to their weight in the initial conditions and/or input signal and the value of the (positive) temporal eigenvalues, while stable modes will decrease and finally vanish. The dynamics of the array can be stopped before any nonlinearity has been reached. In this way a time dependent spatial filter is obtained.

An example of a dispersion curve $K_A(m)$ showing a band of unstable modes is given in Fig.3.

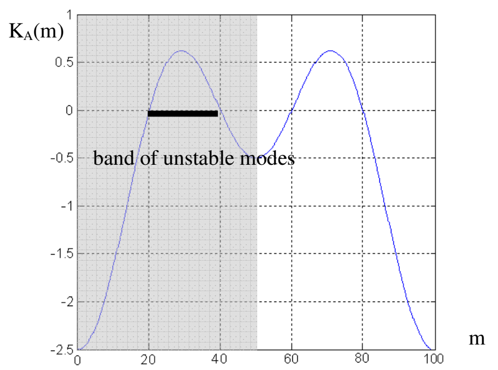

Fig.3. Dispersion curve for A_0=-0.5, A_1=-0.5, A_2=-0.5, M=100 exhibiting a band of unstable modes.

Note that the dispersion curve is significant for values of m less than M/2 as the complex exponential for m>M/2 combine to those with m<M/2 to give cosine functions according to Euler formulas.

For the particular A-template A_0=2 and $A_{\pm 1}$=-1,

$$K_A(m) = 2 - 2\cos\frac{2\pi}{M}m = 4\sin^2\frac{m\pi}{M} \quad (28)$$

a result identical to that obtained in the case of two-grid coupled CNN's where the spatial operator is the discrete Laplacean (Goras and Chua, 1995).

5. CONCLUSION

The spatio-temporal dynamics of analog parallel architectures represents an interesting and intriguing behavior which seems to be worthwhile further studied both from the theoretical and applications points of view. In this work, analytical results concerning the spatio-temporal dynamics of a general class of homogeneous arrays have been presented. They are based on the decoupling technique, which, although intrinsically linear, give significant insight for pattern formation as well, i.e., for the equilibrium points stabilized after the nonlinearities of the cell characteristics have been reached when the cells are piecewise linear or when the dynamics has been stopped before any nonlinearity has been reached. The method puts into evidence the dynamics of the spectrum of the initial conditions with respect to some discrete spatial orthogonal functions dependent on the boundary conditions. The most interesting behavior corresponds to the case when at least one spatial mode is unstable. In certain cases this principle can be used for pattern recognition and feature extraction on the basis of mode competition and spatial selectivity. The dynamics of the spectral components can be studied using the powerful results of linear feedback theory. Preliminary results regarding the effect of the nonidealities of the array parameters show that the dynamics is qualitatively robust.

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