AN ANT COLONY SYSTEM BASED METAHEURISTIC
FOR SOLVING SINGLE MACHINE SCHEDULING PROBLEM

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Abstract: This paper proposes a metaheuristic for solving the Single Machine Scheduling Problem that is implemented by a hybrid system made up of an Ant Colony System and a stochastic descent algorithm called Kangaroo. The hybrid system is based on the collaboration between a social type multiagent system and an Iterated Solution Improvement method.

Keywords: metaheuristics, manufacturing, discrete optimization, stochastic descent, Ant Colony Systems.

1. INTRODUCTION

The main purpose of multiagent systems is the distributed solving of problems. A special type of problem, which can be solved in a distributed way, is the combinatorial optimization problem. The idea of the algorithm "ant system" (Dorigo, *et al.,* 1996) has the source in the study of insects collective behavior. The ants have the capability to act together in order to perform a task, but any of them could not perform alone the task (Beckers, *et al.,* 1992). This is a distributed solving mechanism because every agent has only a very small contribution. The complex collective behavior and the interactions between agents are fundamental in the field of artificial life.

This paper proposes a metaheuristic for solving the Single Machine Scheduling Problem (SMSP). For a given processor and a set of jobs that must be executed on this processor, the problem is to determine the sequence of jobs such that the weighted tardiness (defined in section 2) is minimized. Obviously, because of the combinatorial aspect, this kind of problem is NP-complete. Hence, sub-optimal solutions are generally preferred to optimal ones. A sub-optimal solution is given by an approximation algorithm like genetic algorithm, simulated annealing, tabu search, stochastic descent

algorithms, etc. In paper (Madureira, *et al.*, 2000), an interesting practical resolution is given, in the context of a scheduling system for Dynamic Single Machine Problem. The SMSP is solved using a genetic algorithm and thus, good results are obtained.

In exchange, algorithms like simulated annealing (Kirkpatrick, *et al.*, 1983), tabu search (Gloverf, 1989), stochastic descent (Papadimitriou and Steiglitz, 1982), etc. are *Iterated Solution Improvement* methods, which means that only one solution is improved by an iterative procedure. This kind of methods has abilities to intensify the local search and to detect the local minima. In the last years, hybrid metaheuristics (Vaessens, *et al,* 1992; Taillard *et al.,* 1998; Mahfoud, and Goldberg, 1995) have been developed, giving very interesting results. That is why, this paper proposes an Ant Colony System (ACS) based metaheuristic**,** described in section 3, formed by an ACS and a parallel version of a stochastic descent algorithm, called Kangaroo. The system has the collaborative power of the ACS and the intensification ability of the Kangaroo algorithm (KA).

The paper is organized as follows. In section 2, the Single Machine Scheduling Problem is stated. The general structure of the proposed hybrid system is presented in section 3 and the particularities of an

ACS solving the SMSP are described in section 4. Section 5 outlines the implementation of the Kangaroo Algorithm, whereas the computational results are presented in section 6.heading.

2.   SINGLE MACHINE SCHEDULING PROBLEM

In order to minimize the total Weighted Tardiness for the SMSP the following assumptions are considered: a set of *n* independent jobs (*j= 1,...,n*) is available for processing at time zero and the attributes of the jobs are known in advance. The machine is never kept idle if there are any jobs to complete and it can handle only one job at a time. It processes the jobs without pre-emption. The jobs' set-up times are independent of the jobs' sequence, being included in the job processing times.

For each job *j*, it is considered p_j the processing time, d_j the due date, that means the date when the job should be completed, and w_j the penalty liable for each unit of delay. The jobs' completion starts at time *t=0*. The tardiness of a job is given by

$$T_j = Max\left\{t_j + p_j - d_j, 0\right\},$$

where t_j is the start time of job *j*. The objective function, which will be referred as fitness function, is:

$$f(u) = \sum_{j=1}^{n} w_j T_j$$

where *u* is a solution of the problem, that is a permutation of the job set:

$$u = [\, j_1, j_2, ..., j_i, j_{i+1}, ..., j_n\,], \; j_i \in \left\{1, 2, ..., n\right\}$$

The optimality criterion of the SMSP is $\min_u f(u)$.

This problem is a combinatorial optimization NP-complete problem. The problem can't be solved with deterministic optimal algorithms, as they require a computational time that increases exponentially with the problem size (Garey, and Johnson, 1979).

3.   AN ANT COLONY SYSTEM BASED METAHEURISTIC

In papers (Bauer, *et al.*, 1999; Matthijs, *et al.*, 2000) the Ant Colony System is used to solve SMSP. In this approach, the artificial ants are constructing solutions for this problem and afterwards these solutions are considered initial solutions for a local optimization procedure.

The main idea of the proposed metaheuristic is to use a stochastic descent method instead of the local optimization procedure. In fact, this method is an Iterated Solution Improvement metaheuristic called Kangaroo. As a result, we have a special hybrid metaheuristic based on the collaboration between a social multiagent system - Ant Colony System - and a parallel version of Kangaroo algorithm.

The ACS is made up of N artificial ants which are constructing solutions of the optimization problem. The ants communicate using structured variables whose values represent a provisional quotation of the solutions or of parts of solutions quality. The value of these structured variables simulates the "pheromone" allowing the communication between ants in natural systems. In this case, the structured variables are grouped in a "pheromone" matrix.

The general optimization system is an iterative searching process for a better solution of the given problem. An iteration has two succesive phases. In the first phase, ACS is constructing N solutions and the "pheromone" is continuously updated. The resulting solutions are taken over in the second
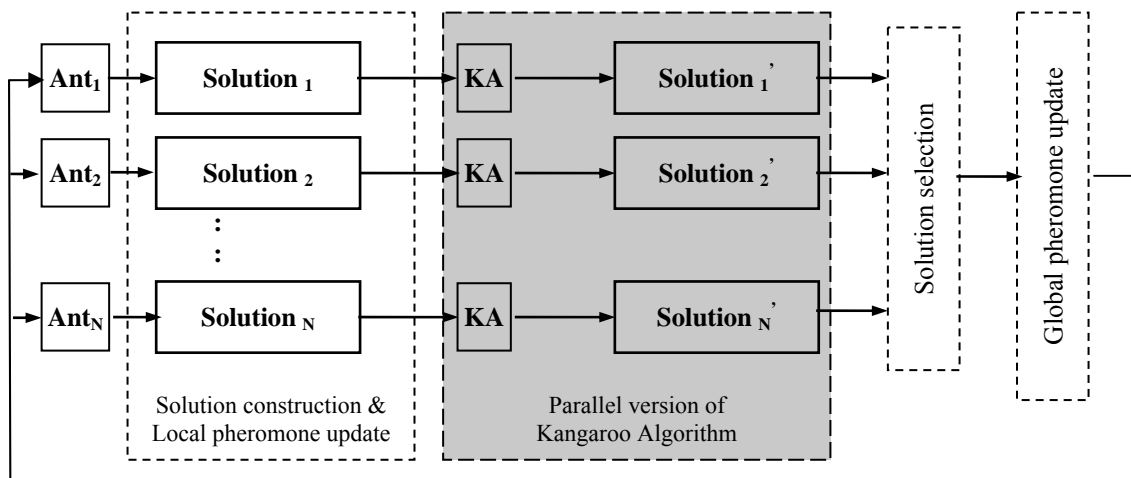

Fig.1 Ant Colony System with Kangaroo algorithm

phase, in order to be improved, by N instances of the KA. From the beginning, the KA makes a local optimization using the solutions produced by each ant as initial solutions. The KA is not limited to this single action, but it keeps trying to improve the current solution following its own strategie. Every instance of the KA is also an iterative procedure looking for a better solution than the current one, in a prescribed number of iterrations. The best found solution is selected and it will be used in a new global "pheromone" updating phase.

In the next iterration, the informations accumulated in the "pheromone" matrix will be used by the ACS, to guide the construction of the new set of solutions. Here after, an outline of the proposed metaheuristic is presented.

Do
1. The Ant Colony System constructs N solutions for the given problem;
2. The parallel version of Kangaroo Algorithm uses the N solutions from the first step as initial solutions and begins N stochastic descent processes. The result is a set of N other better solutions;
3. Select the best solution from this set and use it to update the "pheromone" matrix;
until the *stop criterion.*

Fig. 2. General structure of the proposed metaheuristic

As mentioned before, at the first step, the "pheromone" matrix is also updated during the solutions construction (see section 4). The stop criterion is usually a certain number of iterations.

4. ANT COLONY SYSTEM FOR SOLVING SMSP

Generally speaking, for solving a combinatorial optimization problem ACS needs two kinds of information(Dorigo, *et al.*, 1996; Dorigo, and Gambardella, 1997a; Dorigo, and Gambardella, 1997b). One of them is the heuristic information and the other one is specific to the ACS and concerns the "pheromone".

Each ant of ACS produces a solution of the problem, in the step 1 of the algorithm. This solution is a complete sequence of jobs obtained by an iterative process of placing a job *j* on the position *i* (Matthijs, *et al.*, 2000). At the position *i*, the ant chooses the job *j* meeting two constraints:

a) the job *j* is not already placed in the sequence and

b) the "pheromone" $\tau(i, j)$ has the maximum value for the job *j*.

For SMSP, the "pheromone" $\tau(i,j)$ is a quotation of the interest to place a job *j* on the position *i*. The heuristic information considered by an ant aiming to select a job for the current position may be represented by the inverse of the due date, or the Modified Due Date (MDD)(Bauer, *et al.*, 1999) computed with the formula

$$mdd_j = \max\{d_j, (C + p_j)\},$$

where *C* is the total processing time of the jobs already placed.

An ant *k* will select with probability q_0 the most attractive job, in order to be placed in the current position *i*, that is the task *j* assuring the maximum of $[\tau(i, j)] \cdot [\eta(i, j)]^\beta$. Nevertheless, the same ant may choose with the complementary probability $(1-q_0)$ a job *j* using the probabilistic rule

$$p_k(i, j) = \begin{cases} \dfrac{[\tau(i, j)] \cdot [\eta(i, j)]^\beta}{\sum\limits_{u \in J_k(p)} [\tau(i, j)] \cdot [\eta(i, j)]^\beta} & \text{if } q \in J_k(i) \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where:

- q_0 is a parameter of the algorithm

- $\eta(i, j) = \dfrac{1}{d_j}$ or $\eta(i, j) = \dfrac{1}{mdd_j}$ is the heuristic information

- $J_k(i)$ is the set of not yet placed job by ant *k*;

- β is a parameter which determines the relative importance of heuristic information $(\beta > 0)$.

When all the artificial ants have constructed the solutions, the algorithm uses the following rule for the global updating of the pheromone matrix:

$$\tau(i, j) \leftarrow (1 - \alpha) \cdot \tau(i, j) + \alpha \cdot \Delta\tau(i, j) \quad (2)$$

where

- $\Delta\tau(i, j) = \begin{cases} T^{-1} & \text{if } (i, j) \in \text{best sequence} \\ 0 & \text{otherwise} \end{cases}$

- $0 < \alpha < 1$ is the pheromone increase parameter
- *T* is the total weighted tardiness of the global-best solution from the beginning of the trial.

The application of the rule (2) enforces only the pheromone belonging to the best solution encountered until the current iteration. This is the reason of a premature convergence of the algorithm. Therefore, the rule of local pheromone updating is used. Every time the pheromone information is used by an ant selecting the job *j* for the position *i*, the rule (3) modeling the natural process of pheromone evaporation (forgetting) is applied:

$$\tau(i, j) \leftarrow (1 - \rho) \cdot \tau(i, j) + \rho \cdot \Delta\tau(i, j) \quad (3)$$

where $0 < \rho < 1$ is a parameter.

The term $\Delta\tau(i, j)$ in our implementation is set to τ_0, the initial pheromone level. In order to calculate τ_0, our algorithm constructs an initial solution for SMSP. In this solution, the jobs are placed in increasing order of the due date. The value of τ_0 is initialized with the inverse of the total weighted tardiness of this solution.

5. KANGAROO ALGORITHM

The KA is an approximation technique based on stochastic descent (Fleury, 1995), inspired by the simulation annealing method, but having a quite different searching strategy.

The "Kangaroo" method is implemented by an iterative procedure which minimizes an objective function f(u). A current solution u of the considered problem is replaced by a better one, situated in its neighborhood N(u), using a random selection. The algorithm tries "A" times to improve the current solution, where A is a parameter of the algorithm. If a new improvement is no longer possible, a "*jump*" procedure is performed, in order to escape from the attraction of a local minimum. This time the improvement of the current solution is not compulsory. This procedure can use a different neighborhood definition N'(u).

A detailed description of the KA is given in Minzu, and Henrioud, 1998. The stop criterion is either a maximum iteration number or a bottom bound of the objective function.
The best solution u^* encountered in the iterative process is memorized. At the end of KA, $u*$ is the "optimal" solution proposed by the algorithm.

The neighborhood N(u) is the set of solution u' obtained from u by the permutation of the jobs placed on positions i and $i+1$. For example, if u=[1 4 3 2 5], it holds N(u)={[4 1 3 2 5], [1 3 4 2 5], [1 4 2 3 5], [1 4 3 5 2], [5 4 3 2 1]}.

When a new improvement of the current solution is no longer possible u is replaced by a solution u' given by the "*jump*" procedure. In the case of SMSP a possible definition of the neighborhood N'(u) is the whole search space, but the KA converges (with probability 1) slowly to the global optimum. A very important aspect is the fact that deterministic heuristics may be integrated in "*jump*" procedure, in order to guide the search of an optimum solution keeping the convergence of the KA if the accessibility constraint is met (Minzu, and Henrioud, 1998). That is why, in the case of SMSP the neighborhood N'(u) may be the set of solution u' obtained from u by permutation of the job placed on

i_{max} position, where i_{max} is the position in u of the job j_{max}

$$j_{max} = \arg \max_{j=1,...,n} w_j T_j,$$

with a job placed on position $i<i_{max}$. So the "*jump*" procedure determines the job with the biggest weighted tardiness and replaces it with a job situated on its left. In this way there is a chance to diminish the value of the criterion f(u).

6. IMPLEMENTATION AND COMPUTATIONAL RESULTS

In order to reduce the run time of the hybrid system, the job selection rule is applied on a reduced candidate list that does not contain all the unplaced jobs at the current iteration. This list is updated dynamically for each step and each ant. Every ant keeps a copy of the best found solution until the current iteration. Each time the ant adds a new job j to the current sequence, this job is deleted from the copy of the best found solution. In our implementation, the first 20 jobs, which belong to the best solution and that are not already placed, form the candidate list.

Computational tests were performed in order to compare the proposed hybrid system (ACS+KA)

Table 1 Computational results of the stand-aloneACS

Problem	Optimal value	Best value	Deviation of the Best Value %
Wt100-1	5988	8795	47
Wt100-2	6170	7724	25
Wt100-3	4267	5672	33
Wt100-4	5011	6426	28
Wt100-5	5283	7709	46
Wt100-6	58258	76424	31
Wt100-7	50972	83231	63
Wt100-8	59434	90968	53
Wt50-1	2134	2832	33
Wt50-2	1996	2557	28
Wt50-3	2583	2583	0
Wt50-4	2691	3278	22
Wt50-5	1518	2568	69
Wt50-6	26276	34167	30
Wt50-7	11403	13668	20
Wt50-8	8499	9713	14
Wt40-1	913	913	0
Wt40-2	1225	1431	17
Wt40-3	537	537	0
Wt40-4	2094	2163	3
Wt40-5	990	1090	10
Wt40-6	6955	8151	17
Wt40-7	6324	9083	44
Wt40-8	6865	11474	67

with a stand-alone ACS. The software developed was coded in C and the tests were performed on a PC with 2800 MHz processor.

The two algorithms were applied to the same instances of SMSP. Three sets of 8 problems each with 40 (Wt40-x, x=1,…,8), 50 (Wt50-x, x=1,…,8), and 100 (Wt100-x, x=1,…,8) jobs were considered. Consequently, for both, the stand-alone ACS and the hybrid system, the computational tests were done on a set of 24 instances of the SMSP. These problems were downloaded from the site http://people.brunel.ac.uk/~mastjjb/jeb/orlib/files/, which supplies data and the optimal solution for some Single Machine Scheduling Problems.
The parameters used by the ACS in the two systems are: N=10, q_0=0.9, α=0.9, β=2. The two systems evolved on the same number of iterations. Hence, the stop criterion (see fig. 2) was an upper limit for the general number of iterations. In this case this upper limit was 100.

The results obtained using only the stand-alone ACS for the 3 sets of problems are presented in table 1. The "optimal value" column contains the value of the

optimization criterion for the optimal solution of the problem. For a given instance of SMSP, this value is generally unknown. The "best value" is the value of the optimization criterion for the best solution produced by the stand-alone ACS. The last column gives the deviation of the best value from the optimal one expressed in percents.

The hybrid system ACS+KA ran also over 100 general iterations, but with different values for the number of iterations, denoted M, of the stochastic searching process implemented by KA in each step 2 of the general algorithm (see fig. 2). The results are shown in Table 2.

Four values for M were considered: 1000, 2000, 5000, 10000. The deviation of the best solution from the optimal one is given only for M=5000 and M=10000 iterations. With M=10000 iterations, the hybrid system finds the optimal solution in almost all the cases. Despite the fact this number of iterations doesn't take much time, it is not necessary to adopt such a great number of iterations. Let's remark we are

Table 2 – Computational results with the hybrid system

Problem	Optimal value	Best ACS value	Best value of ACS+KA with:					
			M=1000	M=2000	M=5000	Deviation%	M=10000	Deviation %
Wt100-1	5988	8795	6310	6314	6076	1.47	6215	3.79
Wt100-2	6170	7724	6450	6182	6182	0.19	6182	0.19
Wt100-3	4267	5672	4415	4336	4372	2.46	4297	0.70
Wt100-4	5011	6426	5094	5014	5499	9.74	5069	1.16
Wt100-5	5283	7709	5433	5435	5283	0.00	5367	1.59
Wt100-6	58258	76424	60445	63804	63341	8.72	59845	2.72
Wt100-7	50972	83231	52349	55788	54822	7.55	53063	4.10
Wt100-8	59434	90968	62907	62146	62636	5.39	62817	5.69
Wt50-1	2134	2832	2134	2134	2134	0.00	2134	0.00
Wt50-2	1996	2557	1998	2009	2011	0.75	2008	0.60
Wt50-3	2583	2583	2619	2583	2583	0.00	2583	0.00
Wt50-4	2691	3278	2691	2691	2691	0.00	2691	0.00
Wt50-5	1518	2568	1518	1518	1604	5.67	1518	0.00
Wt50-6	26276	34167	27077	26758	26509	0.89	26403	0.48
Wt50-7	11403	13668	11403	11522	11733	2.89	11403	0.00
Wt50-8	8499	9713	8700	8760	8742	2.86	8700	2.36
Wt40-1	913	913	913	913	913	0.00	913	0.00
Wt40-2	1225	1324	1225	1225	1225	0.00	1225	0.00
Wt40-3	537	573	537	537	537	0.00	537	0.00
Wt40-4	2094	2098	2094	2094	2094	0.00	2094	0.00
Wt40-5	990	1090	990	990	990	0.00	990	0.00
Wt40-6	6955	12949	7024	6955	7055	1.44	6955	0.00
Wt40-7	6324	7087	6636	6324	6437	1.79	6571	3.91
Wt40-8	6865	11015	6919	6881	6919	0.79	6901	0.52

interested in obtaining a good solution and not the optimal one, especially since it is unknown. Because the deviation is satisfactory for M=5000, this number of iterations is recommended for SMSP with 100 jobs.

The same conclusion may to be drawn both for 40 or 50 jobs problems. The value of M may decrease, with very satisfactory results, to 1000 or 2000 iterations.

Comparing the two tables, one can see that the hybrid system is more efficient. Despite the fact the stand-alone ACS evolves during 100 iterations, it doesn't reach the same results as the hybrid system. When M=5000, the price to pay is the very acceptable increasing of the execution time, that means 9, 10 or 17 seconds for 40, 50 or 100 jobs problems, respectively.

7. CONCLUSION

The paper has proposed a metaheuristic for solving SMSP, implemented by a hybrid system made up of an Ant Colony System and a parallel version of the Kangaroo Algorithm.

The KA is a very simple and efficient intensifier that replaces the local optimization proposed in other papers.

The functioning of this hybrid system was compared with a stand-alone ACS. The tests have proven that this structure is more efficient than those of the simple ACS. The number of general iterations and the iterations number of the stochastic descent process are parameters of the algorithm that have to be tuned according to the size of the problem. Very good solutions were found in a quite acceptable time and number of iterations. Moreover, the increasing of the execution time is quite acceptable.

8. REFERENCES

Bauer A., Bullnheimer B., Hartl R.F. And Strauss C. (1999). An ant colony optimization approach for the single machine total tardiness problem, In *Proc. of CEC'99*, pages 1445–1450, IEEE Press, Piscataway, NJ,.

Beckers R., Deneubourg J.L., And Goss S. (1992). Trails and U-turns in the selection of the shortest path by the ant Lasius Niger, *Journal of Theoretical Biology*, **vol. 159**, pp. 397–415.

Dorigo, M., Maniezzo V. And Colorni A., (1996). The Ant System: Optimization by a colony of cooperating agents, *IEEE Transactions on Systems*, Man, and Cybernetics–Part B, **26**, 29–41,

Dorigo M. And Gambardella L.M., (1997a). Ant colonies for the traveling salesman problem, *BioSystems, 43*, 73–81.

Dorigo M. And Gambardella L.M., (1997b). Ant colony system: A cooperative learning approach to the traveling salesman problem, *IEEE Transactions on Evolutionary Computation*, **1**(1):53-66.

Fleury G.(1995). Application des méthodes stochastiques inspirées du recuit simulé à des problèmes d'ordonnancement, *Automatique Productique Informatique Industrielle* vol. **29**-no 4-5, pp. 445-470.

Garey M; Johnson D. (1995). Computers and Intractability: a guide to the theory on NP-completeness- New York W. H. Freeman and Co. Publishers.

Gloverf (1989). Tabu Search- part i. ORSA *Journal of Computing* vol. **1** 1989 pp. 190-206.

Kirkpatrick S. et all, (1983) -Optimization by simulated annealing- *Science* vol.**220** no.4598 May 1983 pp 671-680

Madureira A. Et All (2000) A GA Based Scheduling System for Dynamic Single Machine Scheduling Problem, ISATP 2000, Fukuoka.

Mahfoud S.; Goldberg D, (1995). Parallel recombination simulated annealing: A genetic algorithm. *Parallel computing* vol. **21** 1995 pp. 1-28.

Matthijs Den Besten, Stützle T. And Dorigo M., (2000) Ant Colony Optimization for the Total Weighted Tardiness Problem, In Deb et al, editors, *Proceedings of PPSN-VI, Sixth International Conference on Parallel Problem Solving from Nature*, volume **1917** of LNCS, pages 611-620.

Minzu V. And Henrioud J.M, (1998) Stochastic algorithm for the tasks assignment in single or mixe model assembly lines, *European Journal of Automation*, Vol. **32** No 7-8, pp 831-851.

Papadimitriou C.; Steiglitz K., (1982) Combinatorial Optimization: Algorithms and Complexity - Printice-Hall.

Taillard et al. (1998) La programmation à mémoire adaptative ou l'évolution des algorithmes évolutifs. *Calculateurs parallèles* vol. **10** no. 2 April 1998 pp. 117-140.

Vaessens R. et al (1992) A local search template. Parallel Problem Solving From Nature R. Manner and B. Manderick Edition Université Libre de Bruxelles pp. 67-76 Belgium.