THE INFLUENCE OF NON-HOMOGENOUS DIELECTRIC MATERIAL IN THE WAVEGUIDE PROPAGATION MODES

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Abstract: The aim of this paper is to indicate the equations of electromagnetic wave in homogenous and non-homogenous dielectric material, estabilising the bundary conditions and solves by FEM the equations of the electromagnetic wave in the rectangular cavity. By numeric simulation of the waveguide in the cavity there have been studied the modifications of both the ways of propagation and the field's distribution. The non-homogenous mediums afectes the field's amplitude, obtaining a non-homogenous distribution. Poyting vector of the wave's transmision , indicates the energetic flux's concentration in the air besides the dielectric material

Keywords: Electromagnetic surface waves, waveguide, Dielectric waveguides, Rectangular waveguides

1. INTRODUCTION

The conductors and the dielectric system modes which determine the propagation of electromagnetic waves on the length of the trail are called *waveguides.* These types of waveguides are also called *modes.* They can be electrically and magnetically transversal (TEM), electrically transversal (TE) or magnetically transversal (TM). The horizontal wave is an electrically and magnetically transversal mode (TEM) in which the intensities of the electric and magnetic fields are situated in normal plans on the direction of the propagation. In this case the wave vector is collinear with the direction of movement of the wave and is perpendicular on the size of the electromagnetic field $\overline{k} \cdot \overline{E} = 0$ or $\overline{k} \cdot \vec{H} = 0$ indicating the rotation of the field's size. If the plan of rotation of these sizes does not coincide with the transversal plan on the direction of propagation you obtain the TE and TM modes. In this situation there can have an electric component only in the transversal plan (TE mode) or only the magnetic component in the transversal plan (TM mode)[1]. These last modes exist only if the dielectric medium is homogenous. In the case of non-homogenous modes purely TE or TM this situation does not exist. This situation occurs in many microwave applications such as the dielectric dry. In these applications what is interesting is the way of solving a wave equation.

2. THE MATHEMATICAL MODEL OF THE WAVE EQUATION

The wave equations result from Maxwell's equations[3]. In mediums in which $\rho_v = 0$, **J** = 0 there can exist only movement currents; Maxwell's equations are:

$$\operatorname{rot}\vec{E} = -\frac{\partial \vec{B}}{\partial t}; \operatorname{rot} H = \frac{\partial \vec{D}}{\partial t}; \operatorname{div}\vec{D} = 0; \operatorname{div}\vec{B} = 0 \quad (1)$$

Supposing that the propagation of waves in a rectangular cannel un which the guide is uniform in transversal section (xy) and invariable after the z axis of the propagation. In this hypothesis the sizes of the field from the interior of the guide can be represented as sinusoidal sizes in time and space under the form of:

$$(2) \quad \begin{aligned} \vec{E}(x,y,z,t) &= \vec{E}(x,y)e^{j(\omega t - \gamma z)} \\ \vec{H}(x,y,z,t) &= \vec{H}(x,y)e^{j(\omega t - \gamma z)} \end{aligned}$$

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in which the real values of γ indicate the untenanted propagation on the way of z axis of the mode with the wavelength $\lambda = 2\pi / \gamma$

The form in a complex plan of Maxwell's equations [2] is:

$$\nabla x \underline{\vec{E}} = -j\omega\mu\underline{\vec{H}} \qquad \nabla(\varepsilon\vec{E}) = 0$$
$$\nabla x \vec{H} = j\omega\varepsilon\underline{\vec{E}} \qquad \nabla(\mu\vec{H}) = 0 \quad (3)$$

Dividing the first equation to μ and the second to ε and applying the curl operator there can be determined the wave equation:

$$\nabla x(\frac{\nabla x\vec{E}}{\mu}) = \omega^2\varepsilon\vec{E}$$
$$\nabla x(\frac{\nabla x\vec{H}}{\varepsilon}) = \omega^2\mu\vec{H} \quad (4)$$

a. The homogenous waveguide

In this case the guide put under analysis consists of a metal cavity in the interior of which there is a homogenous and isotropic dielectric. The analysis of the way of propagation of the wave can be simplified by decomposing the sizes of the fields in the sum of two components $\underline{\mathbf{E}}=\underline{\mathbf{E}}_\mathbf{T}+\underline{E}_z\mathbf{e}_z$, a longitudinal one Ez (on the axis of the wave's propagation) and a transversal one $\mathbf{E}_T$ (part of the transversal plan on the direction of the wave), respectively $\underline{\mathbf{H}}=\underline{\mathbf{H}}_\mathbf{T}+\underline{H}_z\mathbf{e}_z$. If the nabla operator [5] is under the form of

$$(5)\ \nabla = \frac{\partial}{\partial x}\vec{e}_x + \frac{\partial}{\partial y}\vec{e}_y - j\gamma\vec{e}_z = \vec{\nabla}_T - j\gamma\vec{e}_z$$

Noted as $-j\gamma = \frac{\partial}{\partial z} = \underline{\gamma}$ then $\frac{\partial^2}{\partial z^2} = -\gamma^2$

and

$$(6)\ \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \gamma^2 = \Delta_T - \gamma^2$$

Where $\vec{\nabla}_T = \frac{\partial}{\partial x}\vec{e}_x + \frac{\partial}{\partial y}\vec{e}_y$ operator in a transversal plan.

Replacing relation (6) in the field of equations (4) there are obtained the bidimensinal equations of the waves for any component of the field:

$$(7)\ \begin{array}{l} \Delta_T\underline{\vec{E}} + (\omega^2\mu\varepsilon - \gamma^2)\underline{\vec{E}} = 0 \\ \Delta_T\underline{\vec{H}} + (\omega^2\mu\varepsilon - \gamma^2)\underline{\vec{H}} = 0 \end{array}$$

The bidimensional equation of the waves attached to the transversal plan can be solved only for the component on the direction of movement Ez respectively Hz. The electromagnetic field's distribution, on perpendicular plan regarding the direction of movement of the wave gives information referring to the ways of propagation. The propagation constant $k^2 = (\omega^2\mu\varepsilon - \gamma^2)$ for which the existing solutions constitute it's own values of the way of propagation supported by the waveguide. For any of it's own value there is an undetermined number of combinations for $\omega,\mu,\varepsilon,\gamma$ which rouses this way.

The dependencies between the transversal and longitudinal components of the field is determined from the first rank dual equations of the electromagnetic fields by replacing relation (5) in equations (3) and separating the components and obtaining:

$$(\omega^2\mu\varepsilon - \gamma^2)E_x = -j(\omega\mu\frac{\partial H_z}{\partial y} + \gamma\frac{\partial E_z}{\partial x})$$
$$(8)\ (\omega^2\mu\varepsilon - \gamma^2)E_y = j(\omega\mu\frac{\partial H_z}{\partial x} - \gamma\frac{\partial E_z}{\partial y})$$
$$(\omega^2\mu\varepsilon - \gamma^2)H_x = j(\omega\varepsilon\frac{\partial E_z}{\partial y} - \gamma\frac{\partial H_z}{\partial x})$$
$$(\omega^2\mu\varepsilon - \gamma^2)H_y = -j(\omega\varepsilon\frac{\partial E_z}{\partial x} + \gamma\frac{\partial H_z}{\partial y})$$

In dielectric homogenous mediums there are two different sets of propagation modes. The first one is determined by the TM mode in which the magnetic field does not have any component on the direction of propagation Hz=0. The second one, the TE mode in which the Ez=o component is null. In these situations the equations (8) are simplified by imposing Hz=o or Ez=0. In the TE mode the bidimensional equation is solved

$$(9)\ \Delta_T\vec{\underline{H}}_z + (\omega^2\mu\varepsilon - \gamma^2)\underline{\vec{H}_z} = 0$$

with the conditions on the boarder of the conductor walls $\vec{n}\vec{H} = 0$ or $\vec{n}(\vec{H}_T + H_z\vec{e}_z) = 0$ implying $\vec{n}\vec{H}_T = 0$ because $\vec{n}\vec{e}_z = 0$. Because the transversal components do not interfere in the solution of the equation by the combination of relations (8) particularly in TE mode

$$(\omega^2\mu\varepsilon - \gamma^2)H_x = j(-\gamma\frac{\partial H_z}{\partial x})$$
$$(\omega^2\mu\varepsilon - \gamma^2)H_y = -j(\gamma\frac{\partial H_z}{\partial y})$$

with $\vec{n}(H_x\vec{e}_x + H_y\vec{e}_y) = 0$ there can be determined the natural condition on the boarder depending on Hz

$$(10)\ \vec{n}(\frac{\partial H_z}{\partial x}\vec{e}_x + \frac{\partial H_z}{\partial y}\vec{e}_y) = \vec{n}(\nabla_T H_z) = 0$$

In TM mode the bidimensional equation

$$(11)\ \Delta_T \underline{\vec{E}}_z + (\omega^2\mu\varepsilon - \gamma^2)\underline{\vec{E}_z} = 0$$

is solved with the conditions on the boarder of the conductor walls $\vec{n}x\vec{E} = 0$. Because $\vec{n}x(\vec{E}_T + E_z\vec{e}_z) = 0$ conducts to $\vec{n}x\vec{E}_T = (\vec{n}x\vec{e}_z)E_z$ which can be solved only if the Ez component is canceled on the field’s frontier, the condition being of Dirichlet type

(12) Ez = 0

b. The non-homogenous guide

Regarding the non-homogenous guides there can be simultaneously the TE and TM mode. The solution to the problem of propagation can come by solving the equations system (4) for E and H wave in which the gradient of permittivity interferes and also the overlapping of solutions. In this case there can appear false ways of propagation as the condition div**B**=0 doesn’t explicitly appear. Another alternative to finishing the problem consists of solving the wave equation for transversal components. The first rank dual equations of the magnetic field in complex $\nabla x\underline{\vec{E}} = -j\omega\mu\underline{\vec{H}}$, $\nabla x\underline{\vec{H}} = j\omega\varepsilon\underline{\vec{E}}$ by replacing the nabla operator(5) and the components of the electric and magnetic field become:

(13)

$$(\vec{\nabla}_T - j\gamma\vec{e}_z)x(\vec{E}_T + \vec{e}_z E_z) = -j\omega\mu(\vec{H}_T + \vec{e}_z H_z)$$

$$(\vec{\nabla}_T - j\gamma\vec{e}_z)x(\vec{H}_T + \vec{e}_z H_z) = j\omega\varepsilon_c(\vec{E}_T + \vec{e}_z E_z)$$

The equation system can be separated and becomes this way:

- The equation of the longitudinal components of the wave:

$$(14)\ \begin{aligned} \nabla_T x\underline{\vec{E}}_T &= -j\omega\mu\underline{H_z}\vec{e}_z \\ \nabla_T x\underline{\vec{H}}_T &= j\omega\varepsilon\underline{E_z}\vec{e}_z \end{aligned}$$

- The equation of the transversal components of the wave:

$$(15)\ \begin{aligned} \vec{\nabla}_T x\vec{e}_z\underline{E}_z - j\gamma\vec{e}_z x\underline{\vec{E}}_T &= -j\omega\mu\underline{\vec{H}}_T \\ \vec{\nabla}_T x\vec{e}_z\underline{H}_z - j\gamma\vec{e}_z x\underline{\vec{H}}_T &= j\omega\varepsilon\underline{\vec{E}}_T \end{aligned}$$

Analyzing the equation of longitudinal components of E type and applying the nabla operator

$$(16)\ \nabla_T x(\frac{\nabla_T x\underline{\vec{H}}_T}{\varepsilon}) = -j\omega(\nabla_T x\underline{E_z}\vec{e}_z)$$

In which from the equation of transversal components (15) the right member is replaced resulting

$$(17)\ \nabla_T x(\frac{\nabla_T x\underline{\vec{H}}_T}{\varepsilon}) = \omega^2\mu\underline{\vec{H}}_T - \gamma\omega(\vec{e}_z x\underline{\vec{E}_T})$$

The form term $\gamma\omega(\vec{e}_z x\underline{\vec{E}_T})$ can be determined from the second relation of the system (15) after the cross with jγ**e**$_z$ and the double division of the cross product obtaining $\omega\gamma(\vec{e}_z x\underline{E}_T) = -\frac{j\gamma\nabla_T \underline{H}_z}{\varepsilon} + \frac{\gamma^2\underline{H}_T}{\varepsilon}$ that replaced in (17) conducts to

$$(18)\ \nabla_T x(\frac{\nabla_T x\underline{\vec{H}}_T}{\varepsilon}) = \omega^2\mu\underline{\vec{H}}_T + \frac{j\gamma\nabla_T \underline{H}_z}{\varepsilon} - \frac{\gamma^2\underline{H}_T}{\varepsilon}$$

From the conservation condition of the magnetic field

$$(19)\ (\nabla_T - j\gamma\vec{e}_z)(\underline{\vec{H}}_T + \underline{H}_z\vec{e}_z) = 0$$

there can be deducted $\nabla_T\underline{\vec{H}}_T = j\gamma\underline{H}_z$ that replaced in (18) conducts to the bidimensional equation of the non-homogenous waves

$$(20)\ \nabla_T x(\frac{\nabla_T x\underline{\vec{H}}_T}{\varepsilon}) = \omega^2\mu\underline{\vec{H}}_T + \frac{\nabla_T(\nabla_T\underline{H}_T)}{\varepsilon} - \frac{\gamma^2\underline{H}_T}{\varepsilon}$$

If the medium is homogenous ε and is constant and the equation (20) is reduced to equation (7). Similarly there can be obtained the wave equation for the transversal components of the intensity of the electric field. The bundary condition can be determined from

$$(21)\ \vec{n}\vec{H} = 0 \text{ and } \vec{n}x\vec{E} = 0.$$

3. THE NUMERIC SIMULATION OF THE FIELD’S DISTRIBUTION IN THE WAVEGUIDE

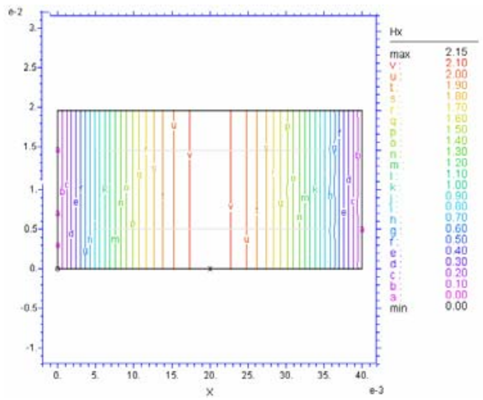
It is considered a rectangular cavity with the dimensions 2x4 cm^2 which contains two dielectric mediums with different permittivity ε_{r1}=1 and ε_{r2}=3,5 (celluloses) according to the figure through which the electromagnetic wave is being propagated if the dielectric material invades the whole cavity when there is a non-homogenous guide. In various applications the dielectric material does not occupy the whole cavity but only an area (central) being surrounded by the dielectric air medium. We propose to solve in the two situations the wave’s equation for the frequency of the microwave applier 2,45 GHz, determining the influence of the non-homogenous medium in it’s propagation. The numeric simulation of the wave’s propagation in the cavity is being done with the finished element method and with the help of PDEase program [4] by simplifying the cross product (20) into two scalar equations corresponding to the Hx, Hy components under the form

$$\frac{\partial}{\partial x}\left(\frac{\partial Hx}{\partial x}+\frac{\partial Hy}{\partial y}\right)\frac{1}{\varepsilon}-\frac{\partial}{\partial y}\left(\frac{\frac{\partial Hy}{\partial x}-\frac{\partial Hx}{\partial y}}{\varepsilon}\right)-\omega^2\mu Hx+\frac{\gamma^2 Hx}{\varepsilon}=0$$

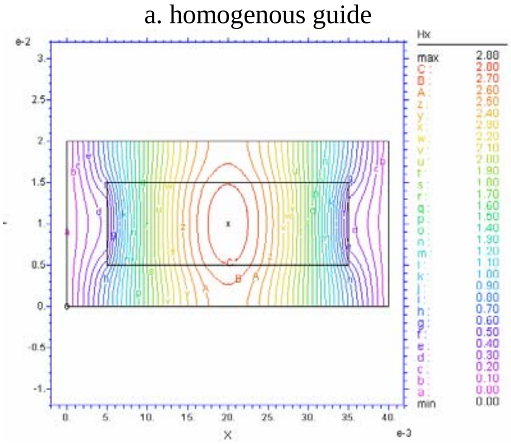
$$\frac{\partial}{\partial y}\left(\frac{\partial Hx}{\partial x}+\frac{\partial Hy}{\partial y}\right)\frac{1}{\varepsilon}+\frac{\partial}{\partial x}\left(\frac{\frac{\partial Hy}{\partial x}-\frac{\partial Hx}{\partial y}}{\varepsilon}\right)-\omega^2\mu Hy+\frac{\gamma^2 Hy}{\varepsilon}=0$$

Completed with the necessary conditions on the boarder $\vec{n}\vec{H}=0$ which implies $\vec{n}(\vec{H}_T+H_z\vec{e}_z)=0$ or $\vec{n}\vec{H}_T=0$ because $\vec{n}\vec{e}_z=0$.Condition $\vec{n}\vec{H}_T=0$ expressed under the $\vec{n}(H_x\vec{i}+H_y\vec{j})=0$ form on the horizontal walls is accomplished if Hy=0 and because $\vec{n}_y\vec{i}=0$ Hx component is expressed through natural (Hx) =0. On the vertical walls the same condition implies Hx=0 respectively natural (Hy) =0. The interference conditions between the dielectric materials are those of conservation of fluxes

The results of the numeric simulation module 1 for the two situations are

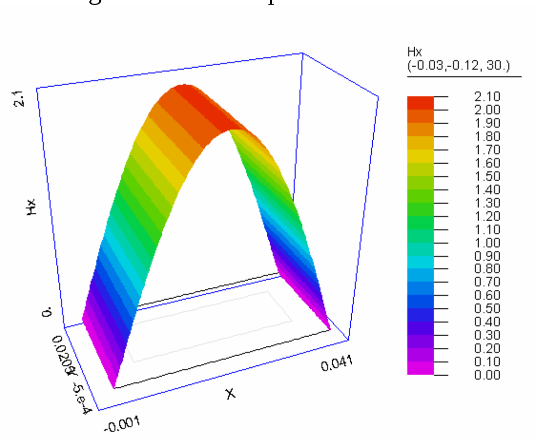


a. homogenous guide

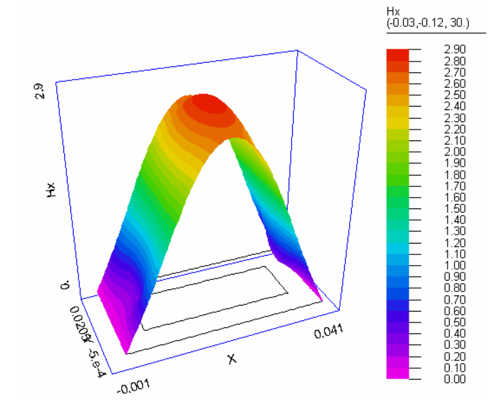


b. non-homogenous guide

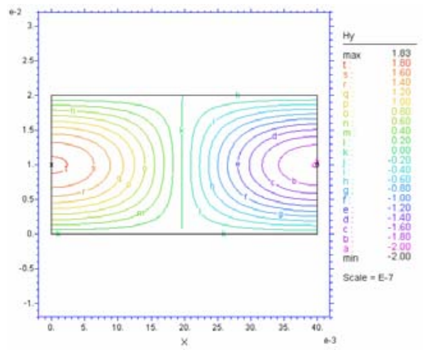
Fig.1. The Hx component distribution



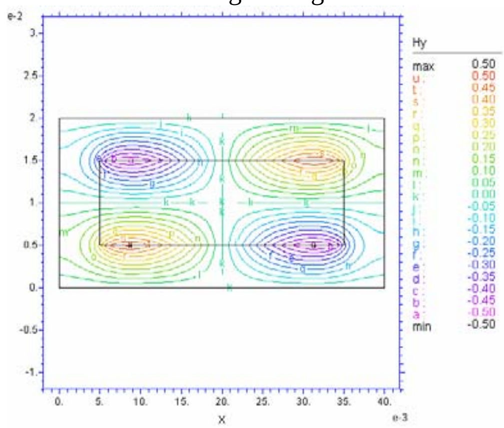
a. homogenous guide



b. non-homogenous guide
Fig.2. Hx surface

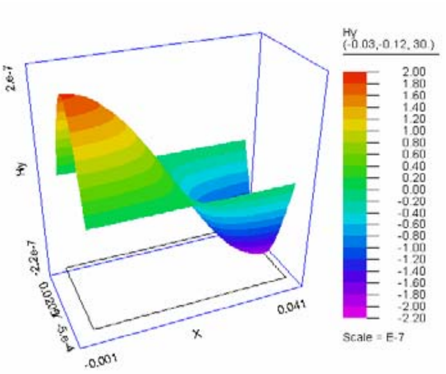


a. homogenous guide

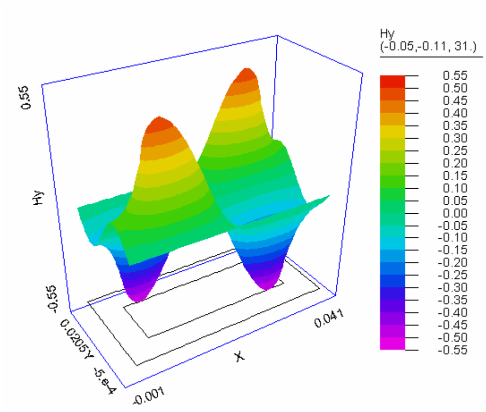


b. non-homogenous guide

Fig.3. Hy component distribution



a. homogenous guide



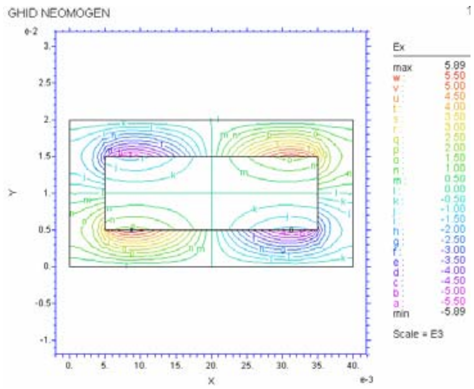
b. non-homogenous guide
Fig.4. Hy surface

The transversal components of the electric field are determinated from the combonation of equation (15) and (19) obtaining

$$Ex = \frac{1}{\omega\varepsilon}\left(\gamma Hy - \frac{\partial}{\partial y}\left(\frac{\frac{\partial Hx}{\partial x}+\frac{\partial Hy}{\partial y}}{\gamma}\right)\right)$$

$$Ey = \frac{1}{\omega\varepsilon}\left(-\gamma Hx + \frac{\partial}{\partial x}\left(\frac{\frac{\partial Hx}{\partial x}+\frac{\partial Hy}{\partial y}}{\gamma}\right)\right) \qquad (22)$$

The distribution of these components for the non-homogenous guide is given in the graphic



The Ex component distribution

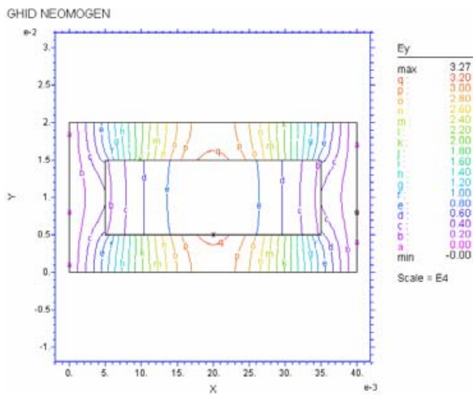


Fig.5. The Ey component distribution

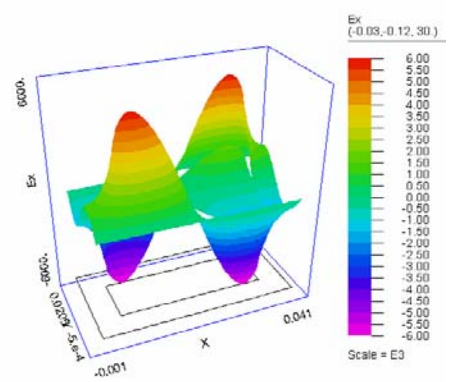


Fig.6. The Ex component surface

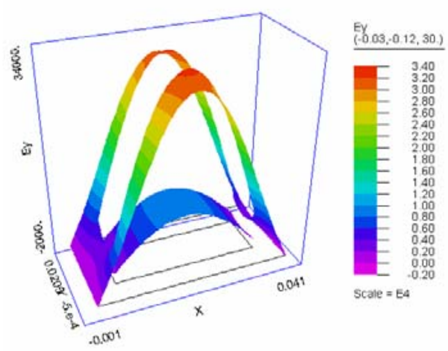
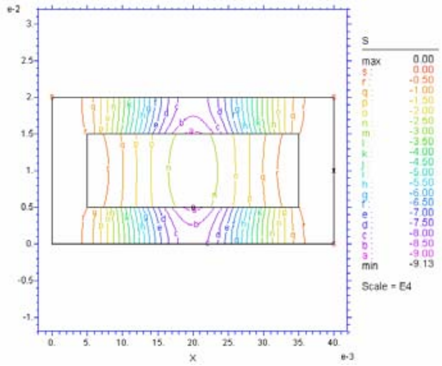


Fig.6. The Ey component surface

Poyting vector of the wave’s transmision, indicate the energetic flux’s concentration in the air besides the dielectric material. This distribution is show in figure 7.



The Poyting distribution

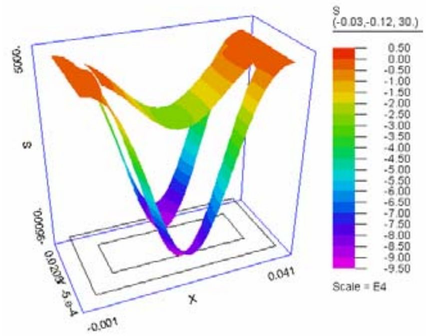


Fig.7. The Poyting surface

4. CONCLUSION

The bidimensional equation of the non-homogenous waves (20) generalizate the equation of homogenous waves and completed with the necessary conditions on the boundary $\vec{n}(H_x\vec{i} + H_y\vec{j}) = 0$ we can determinated the distribution of the field in the cavity. These graphics point out the modification of both the ways of propagation and the field’s distribution. In non-homogenous mediums it is obtained a non-homogenous distribution of the field’s amplitude.

5. REFERENCES

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