SPECTRAL DENSITY CORRECTION OF A SIGNAL AT FREQUENCY VARIABLE TRANSFORMATION

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Abstract: The goal of this paper is to determine analytical expression for the spectral density function of a signal, affected by a known frequency transformation, which do not modify the process energy. Such transformations of frequency variable can frequently appear on spectral density function of a signal, due to physical events (e.g. Doppler effect) or mathematical considerations (e.g. changing the coordinate system). In this case, all components of the spectral density function are modified. The formulas are valid for every spectral component and can be used in signal processing, for model simulation or implementation of advanced algorithm. A case study is illustrated on wave spectrum correction.

Keywords: spectral density, frequency transformation, finite energy, wave spectrum

1. INTRODUCTION

Many applications, in fields like radar, sonar, marine systems, biomedical engineering, are based on signal detection with unknown parameters, such as phase and frequency. Frequently, known transformations of frequency variable can appear on spectral density function of a signal, without changing the process energy, due to physical events (e.g. Doppler effect) or mathematical considerations (e.g. changing the coordinate system). In this case, all components of the spectral density function are modified.

For example, in pulsed Doppler radar systems, the frequency of the back scattered pulse is not the same as the one of the transmitted pulse. In this case, only one spectral component is useful and the frequency transformation is linear (Lyman, 2003). Without knowing Doppler frequency, it is difficult to carry out target detection, prior to frequency estimation. If the dominant type of interference is clutter that is correlated and not necessarily Gaussian, the detection problem becomes even more difficult (Ong and Zoubir, 2003). For non-Gaussian interference, the clutter models include Weibull, K, Rayleigh mixture (Sangston and Gerlach, 1999).

Another example of frequency transformation is the relative frequency between the wave and the ship, which represents the wave's frequency in the coordinate system with the origin on the ship's center of gravity. The relative frequency between the wave and the ship modifies the wave spectrum and this transformation must be taken into account for wave model generation (Nicolau, 2004). This is a more complex frequency transformation than Doppler effect, because the wave propagation is done, not into a specific environment, but at the separation surface between two different environments.

The wave, which is regarded as an ergodic random process with elevation $\zeta(t)$ and zero mean, has for every spectral component a forward speed, which is not constant and depends on the component frequency. As a result, the frequency transformation is not linear, and in many cases it is not even global bijection (Nicolau and Ceanga, 2001).

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The goal of this paper is to determine analytical expression for the spectral density function of a signal, affected by known frequency transformation, which do not modify the process energy. The formulas are valid for every spectral component and can be used in signal processing, for model simulation or implementation of advanced algorithm.
The paper is structured as follows. In section 2, preliminaries and mathematical models are discussed. Analytical corrections of spectral density function at frequency transformation are determined in section 3. Section 4 presents a case study, referring to regular waves, which are the main disturbances for surface vessels. Simulation results are pointed out in section 5, and conclusions are presented in section 6.

2. PRELIMINARIES AND MATHEMATICAL MODELS

The power spectral density function $S_{xx}(\omega)$ of an ergodic random process with finite energy $x(t)$ is a measure of the process energy. Considering $x_k(t)$ a representative instance of the ergodic process $x(t)$ into finite time interval T, as shown in Fig. 1, the expression of the spectral density function can be determined. The energy of $x_k(t)$ instance is:

$$E = \int_{-\infty}^{\infty} x_k^2(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X_k(\omega)|^2\,d\omega, \qquad (1)$$

where $X_k(\omega)$ is Fourier transform of $x_k(t)$.

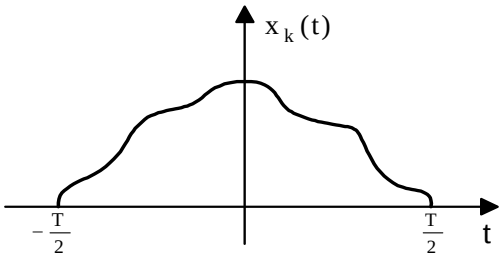

Fig. 1. Instance $x_k(t)$ of the ergodic process $x(t)$

The average power on time interval T is:

$$P_T = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{|X_k(\omega)|^2}{T}\,d\omega \qquad (2)$$

From (2), the average power of $x_k(t)$ results:

$$P = \langle x_k^2(t)\rangle = \lim_{T\to\infty} P_T = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lim_{T\to\infty} \frac{|X_k(\omega)|^2}{T}\,d\omega \qquad (3)$$

By definition, the average power of an ergodic random process is the temporal mean square value of process's representative instance $x_k(t)$, and it depends on power spectral density function $S_{xx}(\omega)$:

$$P = \langle x_k^2(t)\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{xx}(\omega)\,d\omega \qquad (4)$$

Identifying the terms in (3) and (4), it results:

$$S_{xx}(\omega) = \lim_{T\to\infty} \frac{|X_k(\omega)|^2}{T} \qquad (5)$$

Knowing that the power spectral density is even and real function, $S_{xx}(\omega) = S_{xx}(-\omega)$, the average power of the ergodic random process can be written:

$$P = \langle x_k^2(t)\rangle = \frac{1}{\pi}\int_0^{\infty} S_{xx}(\omega)\,d\omega = \int_0^{\infty} \phi_{xx}(\omega)\,d\omega \qquad (6)$$

where $\phi_{xx}(\omega)$ is the mean square spectral density function of the representative instance of ergodic process, shortly called spectral density function, which is used in this paper. If the process energy is finite, then the average power is also finite and the integrals in (1) and (6) are bounded.

3. CORRECTION OF SPECTRAL DENSITY FUNCTION

Consider a process of finite energy, such as a signal, and let $f(\omega)$ be its spectral density function, $f: X \to [0, \infty)$, $X=[0,\infty)$. Due to finite process energy, it results:

$$\int_0^{\infty} f(\omega)\,d\omega < M, \quad M \in \Re^+ \qquad (7)$$

Let g function be a known transformation of frequency variable, $g: X \to Y$, $Y=[0, \infty)$, $y=g(\omega)$. The y and ω variables are called corrected frequency and initial frequency, respectively (Nicolau and Ceanga, 2001). Due to this transformation, the spectral density function $f(\omega)$ in (7) is transformed into another unknown function $h(y)$, $h : Y \to [0, \infty)$, which is called in this paper the corrected spectral density function, so that the energy and average power of the process to remain unchanged:

$$I_1 = \int_0^{\infty} f(\omega)\,d\omega = I_2 = \int_0^{\infty} h(y)\,dy < M, \quad M \in \Re^+ \qquad (8)$$

The shape of the corrected-spectral-density function h and the corrected frequency range corresponding to non-zero spectral components depend on frequency transformation. The goal is to obtain the analytical expression of h function. Suppose the g function is more general, being not global but local bijection, with n distinct zones, as shown in Fig. 2.
It can be observed that specific values of corrected frequency (y) can be obtained by g transformation from different initial frequency components (ω). In this case, the component of corrected spectral density function is generated by several components of initial spectral density function. The g function is defined:

$$g: \bigcup_{i=1}^{n} X_i \to \bigcup_{i=1}^{n} Y_i, \quad g(\omega) = \begin{cases} g_1(\omega) & \omega \in X_1 \\ ----- & \\ g_n(\omega) & \omega \in X_n \end{cases}, \qquad (9)$$

where $g_i : X_i \to Y_i$ are bijections, $i = 1\ldots n$.

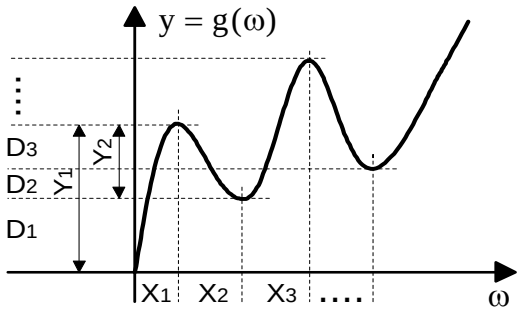

Fig. 2. Frequency transformation $g(\omega)$

The subsets X_i represent a covering for the function domain X, and they are distinct, except eventually for their extremities. The subsets Y_i represent a covering for the codomain Y, not necessary to be distinct:

$$\bigcup_{i=1}^{n} X_i = X\,, \qquad \bigcup_{i=1}^{n} Y_i = Y \tag{10}$$

For every bijection g_i, the inverse function exists:

$$g_i^{-1}: Y_i \to X_i\,, \quad g_i^{-1}(y) = \omega \in X_i\,, \forall\, y \in Y_i \tag{11}$$

It can be observed that:

$$d\omega = \left(\left(g_i^{-1}\right)'(y)\right)dy\,, \quad |d\omega| = \left|\left(g_i^{-1}\right)'(y)\right| \cdot |dy| \tag{12}$$

$$\text{where } (.)'(y) = \frac{d}{dy}(.) \text{ and } \left(g_i^{-1}\right)'(y) = \frac{1}{g_i'(\omega)} \tag{13}$$

On local extreme points of g function, it results:

$$g_i'(\omega) = 0 \quad \Rightarrow \quad \left(g_i^{-1}\right)'(y) \to \infty \tag{14}$$

Using (10), the first integral in (8) results:

$$I_1 = \int_0^{\infty} f(\omega)\, d\omega = \sum_{i=1}^{n}\left(\int_{X_i} f(\omega)\, d\omega\right) \tag{15}$$

Changing the variable, $\omega = g^{-1}(y)$, (15) becomes:

$$I_1 = \sum_{i=1}^{n}\left(\int_{Y_i} f\left(g_i^{-1}(y)\right)\cdot\left|\left(g_i^{-1}\right)'(y)\right|\cdot dy\right), \tag{16}$$

where the absolute value is considered, taking into account the integration sense.

Let it be a new covering for codomain, with smaller distinct subsets D_j, $j = 1\ldots m$, as shown in Fig. 2:

$$Y = \bigcup_{i=1}^{n} Y_i = \bigcup_{j=1}^{m} D_j\,, \tag{17}$$

$$D_{j1} \bigcap D_{j2} = \Phi, \quad \forall\, j_1, j_2 = \overline{1,m}\,, \quad j_1 \neq j_2 \tag{18}$$

Subsets D_j are small enough to be included into Y_i:

$$Y_i \bigcap D_j = \begin{cases} D_j & D_j \subseteq Y_i \\ \Phi & D_j \not\subset Y_i \end{cases} \tag{19}$$

As a result, every subset Y_i can be written using different subsets D_j. To extend the number of subsets included in every subset Y_i to m, new theoretical subsets A_{ji} can be used:

$$Y_i = \bigcup_{j=1}^{m} A_{ji}\,, \quad A_{ji} = \begin{cases} D_j & D_j \subseteq Y_i \\ \Phi & D_j \not\subset Y_i \end{cases} \tag{20}$$

Using (20) in (16), the integration domain defined by subsets Y_i is replaced with a new domain, which includes subsets D_j, resulting:

$$I_1 = \sum_{i=1}^{n}\left(\sum_{j=1}^{m}\left(\int_{A_{ji}} f\left(g_i^{-1}(y)\right)\cdot\left|\left(g_i^{-1}\right)'(y)\right|\cdot dy\right)\right) \tag{21}$$

The integration domain can be specified with subsets D_j, using additional coefficients:

$$a_{ji} = \begin{cases} 1 & A_{ji} = D_j \\ 0 & A_{ji} = \Phi \end{cases} \quad i = \overline{1,n} \quad j = \overline{1,m} \tag{22}$$

Using (22) in (21), the integral in (8) results:

$$I_1 = \sum_{i=1}^{n}\left(\sum_{j=1}^{m}\left(\int_{D_j} a_{ji}\cdot f\left(g_i^{-1}(y)\right)\cdot\left|\left(g_i^{-1}\right)'(y)\right|\cdot dy\right)\right) \tag{23}$$

Reordering the terms, the first integral becomes:

$$I_1 = \sum_{j=1}^{m}\left(\int_{D_j} \sum_{i=1}^{n} a_{ji}\cdot f\left(g_i^{-1}(y)\right)\cdot\left|\left(g_i^{-1}\right)'(y)\right|\cdot dy\right) \tag{24}$$

On the other hand, the second integral in (8) is:

$$I_2 = \int_0^{\infty} h(y)\, dy = \sum_{j=1}^{m}\left(\int_{D_j} h(y)\, dy\right) \tag{25}$$

In (8), the equality of the integrals must be true for every corrected-frequency value y. This implies in (24) and (25), the equality of the integrals for every subset D_j and also the equality of the functions under integrals. Identifying terms in (24) and (25), results:

$$h: \bigcup_{j=1}^{m} D_j \to [0,\infty)\,, \quad h(y) = \begin{cases} h_1(y) & y \in D_1 \\ ----- & \\ h_m(y) & y \in D_m \end{cases}, \tag{26}$$

where

$$h_j : D_j \to [0,\infty)\,,$$

$$h_j(y) = \sum_{i=1}^{n} a_{ji}\cdot f\left(g_i^{-1}(y)\right)\cdot\left|\left(g_i^{-1}\right)'(y)\right| \tag{27}$$

In (27) it can be observed that, for every corrected-frequency value y, the corrected spectral component $h_j(y)$ is a sum of maximum n components from initial spectral density function, which are modified based on frequency transformation. Also, for every local extreme of g function, which are extremities of subsets D_j, h function becomes discontinuous and unbounded, but the integral remains bounded.

Considering only a part of the process energy, corresponding to spectral components generated by frequencies in the specified range $[\omega_1, \omega_2]$, it results:

$$\int_{\omega_1}^{\omega_2} f(\omega)\, d\omega = \int_{y_1}^{y_2} h(y)\, dy \tag{28}$$

where $y_1 = g(\omega_1)$ and $y_2 = g(\omega_2)$.

4. CASE STUDY

4.1. Relative Frequency of a Wave

The waves in open sea, which are main disturbances for surface vessels, can be regarded as an ergodic random process with elevation $\zeta(t)$ and zero mean. For knowing the statistical parameters of the wave and generating the wave model, the mean square spectral density function $\phi_{\zeta\zeta}(\omega)$ of the wave elevation $\zeta(t)$, shortly called wave spectrum, must be known.
The relative frequency of a wave (ω_r) represents the wave's frequency in the coordinate system with the origin on the ship's center of gravity. It depends on the speed of the ship (v) and the incidence angle (γ), which is the angle between the heading and the direction of the wave, as illustrated in Fig. 3.

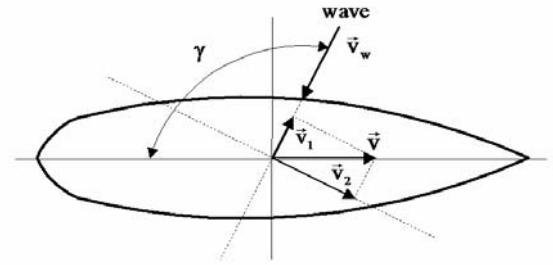

Fig. 3. Incidence angle γ modifies the relative speed between the wave and the ship

The relative frequency between the wave and the ship modifies the wave spectrum and this transformation must be taken into account for wave model generation.
If the ship's speed v is non-zero then the component on the wave direction v_1 is non-zero, which modifies the relative speed v_r and the relative frequency of the wave ω_r. The relative frequency of the wave ω_r is given by the following formula :

$$\omega_r = g(\omega) = \omega \cdot \left(1 - \frac{\omega \cdot v \cdot \cos\gamma}{g} \right) \tag{28}$$

Equation (28) can be written with one parameter:

$$\omega_r = \omega \cdot (1 - a \cdot \omega) \tag{29}$$

where the parameter a has the expression:

$$a = \frac{v \cdot \cos\gamma}{g} \tag{30}$$

The sign of the relative frequency shows the direction that the wave hit the ship and it is not important in studying the influence of the wave over the ship (Nicolau, 2004). Hence, absolute value can be taken in (29), resulting:

$$\omega_r = g(\omega) = \omega \cdot |1 - a \cdot \omega| \tag{31}$$

The range of values for the parameter $a \in [a_{min}, a_{max}]$ depends on the maximum value of ship speed v_{max}:

$$a_{\min} = -\frac{v_{\max}}{g} < 0\,, \quad a_{\max} = \frac{v_{\max}}{g} > 0 \tag{32}$$

For waves in open sea, (31) is true for every component of the wave spectrum.

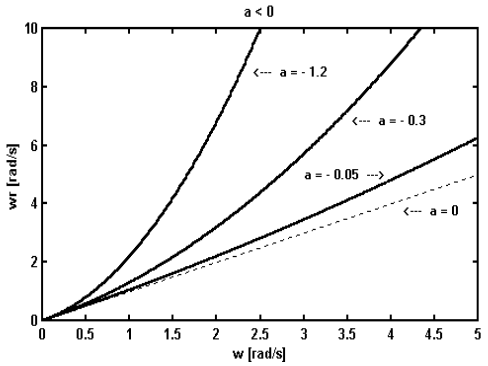

Fig. 4. Variation of relative frequency with a<0

If $a=0$ the frequency remains unchanged $\omega_r=\omega$, as illustrated with dotted line in Fig. 4.
If $a<0$ the expression of relative frequency in (31) is:

$$\omega_r = \omega \cdot (1 - a \cdot \omega) > \omega\,, \tag{33}$$

and the function is a global bijection. The relative frequency variation with parameter $a<0$ is shown in Fig. 4, for three negative values: -0.05, -0.3, -1.2.
If $a>0$ the expression of relative frequency is:

$$\omega_r = g(\omega) = \begin{cases} \omega \cdot (1 - a \cdot \omega) & \omega \le \dfrac{1}{a} \\ \omega \cdot (a \cdot \omega - 1) & \omega > \dfrac{1}{a} \end{cases} \tag{34}$$

The function is not a global but local bijection, as shown in Fig. 5, which represents the variation of relative frequency with $a>0$ for three positive values: 0.05, 0.3 and 1.2..

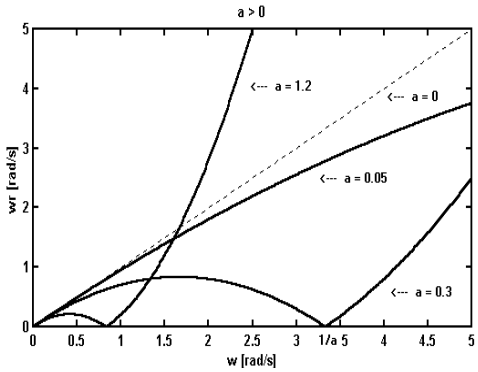

Fig. 5. Variation of relative frequency with a>0

There are frequencies below and above the dotted line, which means that for some $\omega_r \le \omega$ and for the other $\omega_r > \omega$. The speed of wave components with $\omega<1/a$ is greater than the ship's speed and those with $\omega>1/a$ have speed smaller than the speed of the ship.

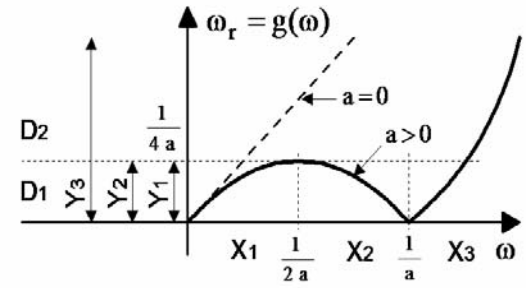

Fig. 6. The subsets of domain and codomain of ω_r

The relative frequency with *a>0* is a continuous function with three local bijections, shown in Fig. 6:

$$\omega_r = \begin{cases} \omega \cdot (1 - a \cdot \omega) = g_1(\omega) & \omega \in X_1 \\ \omega \cdot (1 - a \cdot \omega) = g_2(\omega) & \omega \in X_2 \\ \omega \cdot (a \cdot \omega - 1) = g_3(\omega) & \omega \in X_3 \end{cases} \quad (35)$$

The relative frequency with *a>0* and all the subsets defined in previous section are illustrated.

4.2. Wave Spectrum Correction

To analytically describe a wave spectrum, there are defined several similar formulas for fully developed waves (Price and Bishop, 1974), of the form:

$$\phi_{\zeta\zeta}(\omega) = \frac{A}{\omega^5} \cdot e^{-\frac{B}{\omega^4}} \quad (36)$$

The parameters A and B have different forms, as function of the wind speed or the significant wave height $h_{1/3}$ and the average period. For example, ITTC spectrum has (Pierson and Moskowitz, 1964):

$$A = 0.7795, \qquad B = 3.11/h_{1/3}^2 \quad (37)$$

All spectral components of the wave are modified by ω_r and form a new corrected spectrum, whose expression can be obtained based on energy equality:

$$\int_0^\infty \phi_{\zeta\zeta}(\omega)\, d\omega = \int_0^\infty \phi_{\zeta\zeta\, r}(\omega_r)\, d\omega_r \quad (38)$$

The m_0 moment represents the mean square value of wave elevation ζ(t), being a wave characteristic:

$$m_0 = \int_0^\infty \phi_{\zeta\,\zeta}(\omega)\, d\omega = \langle \zeta^2(t) \rangle = \frac{A}{4 \cdot B} \quad (39)$$

Also m_0 represents the mean value of the power for the wave concerned. Hence, the energy on the wave surface unity (E_v) will be proportional with m_0. Values of *a* from (30) affect the expression and shape of the corrected spectrum.

If *a=0* the wave spectrum remains unchanged $\phi_{\zeta\zeta\, r}(\omega_r) = \phi_{\zeta\zeta}(\omega)$. If *a<0* then *h(y)* has a single expression and the corrected wave spectrum is:

$$\phi_{\zeta\zeta\, r}(\omega_r) = \frac{\phi_{\zeta\zeta}\left(\dfrac{1 - \sqrt{1 - 4 \cdot a \cdot \omega_r}}{2 \cdot a}\right)}{\sqrt{1 - 4 \cdot a \cdot \omega_r}} \quad (40)$$

This is the most used case in simulations, as the ship and the wave have opposite movements. The spectrum is illustrated in Fig. 7, being bounded for any value of *a*. While *a* values moves to negative values, the spectrum moves to high frequencies.

If a>0, the expression of the corrected spectrum is:

$$\phi_{\zeta\zeta\, r}(\omega_r) = \begin{cases} \phi_{\zeta\zeta}(1/a) & \omega_r = 0 \\ \sum_{i=1}^{3} \phi_{\zeta\zeta\, ri}(\omega_r) & \omega_r \in (0, 1/4a] \\ \phi_{\zeta\zeta\, r3}(\omega_r) & \omega_r \in (1/4a, \infty) \end{cases} \quad (41)$$

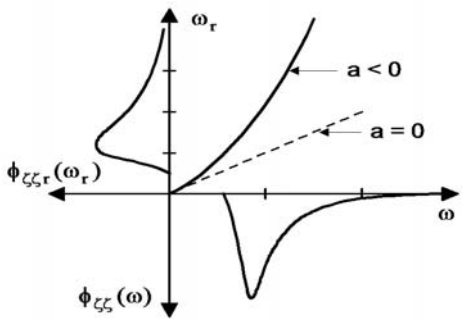

Fig. 7. Transformation of the wave spectrum for a<0

The expressions of $\phi_{\zeta\zeta\, ri}(\omega_r)$ are:

$$\phi_{\zeta\zeta\, r1}(\omega_r) = \frac{\phi_{\zeta\zeta}\left(\dfrac{1 - \sqrt{1 - 4 \cdot a \cdot \omega_r}}{2 \cdot a}\right)}{\sqrt{1 - 4 \cdot a \cdot \omega_r}} \quad (42)$$

$$\phi_{\zeta\zeta\, r2}(\omega_r) = \frac{\phi_{\zeta\zeta}\left(\dfrac{1 + \sqrt{1 - 4 \cdot a \cdot \omega_r}}{2 \cdot a}\right)}{\sqrt{1 - 4 \cdot a \cdot \omega_r}} \quad (43)$$

$$\phi_{\zeta\zeta\, r3}(\omega_r) = \frac{\phi_{\zeta\zeta}\left(\dfrac{1 + \sqrt{1 + 4 \cdot a \cdot \omega_r}}{2 \cdot a}\right)}{\sqrt{1 + 4 \cdot a \cdot \omega_r}} \quad (44)$$

The shape of the corrected spectrum $\phi_{\zeta\zeta r}(\omega_r)$ is changing with *a*, depending where the two particular frequencies $\omega = 1/(2a)$ and $\omega = 1/a$ are placed, inside or outside of the range $[\omega_1, \omega_2]$ of the initial spectrum. Four situations are possible, with bounded and unbounded corrected spectra. The two situations with unbounded spectra are represented in Fig. 8.

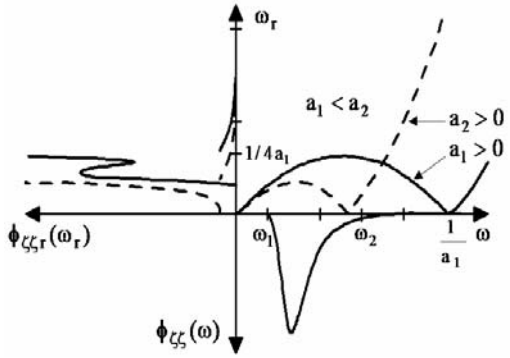

Fig. 8. Transformation of the wave spectrum for $a \in [1/(2\omega_2), 1/\omega_1]$

If $1/(2a) \le \omega_2 < 1/a \iff a \in [1/(2\omega_2), 1/\omega_2)$, then the corrected spectrum becomes discontinuous and unbounded for relative frequency $\omega_r = 1/(4a)$, being illustrated with continuous line in Fig. 8. The corrected spectrum has a predominant spectral component at frequency $\omega_r = 1/(4a)$.

If $\omega_1 \le 1/a \le \omega_2 \iff a \in [1/\omega_2, 1/\omega_1]$, in addition to the discontinuous point for $\omega_r = 1/(4a)$, the spectral component for $\omega_r = 0$ is not zero (dotted line), which corresponds to the situation when the ship is siting on and moving with the wave.

5. SIMULATION RESULTS

For simulations, ITTC wave spectra and a ship with $v_{max}=12$ *m/s* are considered. The limits of parameter *a*, computed with (32), are: $a_{min}= - 1.3$, $a_{max}=1.3$.
If $a<0$ then *a* can take any negative value in the range *[-1.3, 0]* and the corrected wave spectrum with the analytical expression in (40) remains bounded.
The wave spectrum with significant height $h_{1/3}=5$ *m* (dotted line), and the corrected wave spectra for three values of *a*: -0.2, -0.6 and –1.3 are shown in Fig. 9.

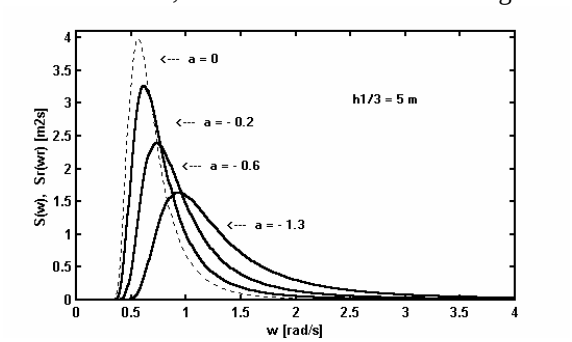

Fig. 9. Wave spectrum correction for $h_{1/3} = 5$ *m*

In general, the wave model takes into account only a limited frequency band, which contents the spectral components with the most part of the wave energy.
For example, if $0<a<1/(2\omega_2)$ the initial wave spectrum is placed to the left of $\omega=1/(2a)$, as shown in Fig. 10.

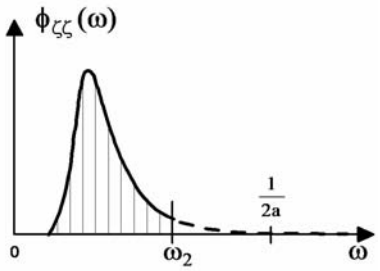

Fig. 10. Initial wave spectrum for $a < 1/(2\omega_2)$

The frequency ω_2 is chosen so that the most part of the wave energy to be contained in the wave components generated by the frequencies $\omega \le \omega_2$:

$$\int_0^{\omega_2} \phi_{\zeta\zeta}(\omega)\, d\omega \;=\; p \;\cdot \int_0^{\infty} \phi_{\zeta\zeta}(\omega)\, d\omega\,, \qquad (45)$$

where the second integral is equal to the zero moment of the wave spectrum, $m_0=A/(4\cdot B)$, and represents the mean value of the wave power on the surface unity. In this case, the limits of parameter *a* become: $0<a<a_2$. The a_2 and ω_2 values are computed in (Nicolau and Ceanga, 2001), based on weight $(0<p<1)$ chosen to approximate the wave energy:

$$a_2 = \frac{1}{2}\cdot \sqrt[4]{\frac{-\ln(p)}{B}}\,, \quad \omega_2 = \sqrt[4]{\frac{B}{-\ln(p)}} \qquad (46)$$

For $h_{1/3}=1m$, the a_2 variations with *p* are shown in Table I. Selecting the zone $p=0.95$, the ranges for *a* will be *(0, 0.18)*. The limit ω_2 for p=0.95 is ω_2 =2.79.

Table I. Variations of a1 and a2 with p

p	0.99	0.95	0.9	0.85	0.8
a_2	0.12	0.18	0.21	0.24	0.26

In Fig. 11 are illustrated the initial wave spectrum (dotted line) and the corrected wave spectra for two values of $a < 0.18$: 0.08 and 0.14. From (31), the limits ω_{2r} with *a* indicated above are: 2.17 and 1.7.

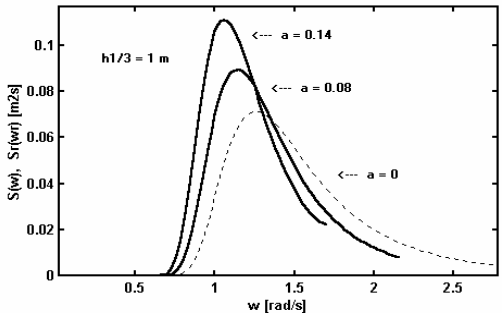

Fig. 11. Corrected wave spectra for $a \in (0, 0.18)$

6. CONCLUSIONS

The analytical expressions for the spectral density function of a signal, affected by a known frequency transformation, which do not modify the process energy, are determined. The formulas are valid for every spectral component. Every corrected spectral component $h_j(y)$ is a sum of maximum *n* components from initial spectral density function, which are modified based on frequency transformation.

REFERENCES

Lyman, R. J. (2003). Optimal mean-square prediction of the mobile-radio fading envelope. *IEEE Trans. Signal Processing*, **51**, no.3, pp. 819-824.

Ong, H. T. and A. M. Zoubir (2003). Bootstrap-based detection of signals with unknown parameters in unspecified correlated interference. *IEEE Trans. Signal Processing*, **51**, no.1, pp. 135-141.

Sangston, K. J. and K. R. Gerlach (1999). Coherent detectionof radar targets in a non-Gaussian background. *IEEE Trans. Signal Processing*, **47**, no.9, pp. 2538-2541.

Nicolau, V. (2004). Contributions on advanced control of naval systems. *Ph.D. dissertation*, Dept. Aut. Control, Dunarea de Jos University of Galati, Galati.

Nicolau, V. and E. Ceanga (2001). Wave Spectrum Correction with the Ship's Speed and the Incidence Angle. *IFAC Conf. on Control Applications in Marine Systems (CAMS 2001)*, pp. 331-336, Glasgow, UK, ISBN 0-080-43236-0.

Price, W.G. and R.E.D. Bishop (1974) *Probabilistic theory of ship dynamics.* Chapmann and Hall.

Pierson, W.J. and L. Moskowitz (1964). A proposed spectral form for fully developed wind seas based on the similarity theory of S.A. Kitaigorodskii. *Journal of Geophysical Research*, **69**, pp. 24-26.