1. Introduction

Multiple equation econometric models are currently the main tool in management - at the micro or macro level - for a more rigorous and not intuitive or descriptive decision. In this sense such models are frequently used to explain the activities, simulation and in particular to estimate the likely evolution of economic phenomena. An issue that remains open concerns the degree similarity (relevance or compatibility) of the structure model with simultaneous equations and models developed recursive structure of processes and economic systems which they describe. In this case, after checking the significance of the estimators, the ratio of correlation and the model shows that it provides a meaningful description of the development of the region's economy for the period. This paper develops the theoretical aspects of the econometric model and a case study using multiple equations model with dynamic Keynesian model.

2. Theoretical aspects of the econometric model

Isomorphic model is a representation of reality [7], it offers an intuitive picture, but for the purposes of rigorous logical structure, the phenomenon studied and ties that facilitate the discovery of regularities impossible or very difficult to determine in other ways. The model is a representation of the system and how it works. In the modelling process is the existence of an analogy between the modelled entity (system, subsystem) of reality and model. The model is actually realized the representation of the system like a lot of parts interacting one with another [1].

If we consider the set of all objects \( \{O\} \), where we define three subsets: N - subsets of natural or social, A - subset of physical objects constructed by humans and C - a subset of conceptual objects (concepts, scientific theories), shall be deemed that every element \( x \in O \) is analogous to another element \( y \in O \) if:

a) \( x \) and \( y \) have the common property or even identical;

b) There is a correspondence between the parties or between \( x \) and \( y \) properties of these parts. Generally, the relationship of analogy between \( x \) and \( y \), for which the symbol \( x \sim y \) is used, has the following properties (Bunge)

- is symmetric (\( x \sim y \) \( \Rightarrow \) (\( y \sim x \));
- is reflexive (\( x \sim x \));
- the relationship of transitivity it is not generally valid:

If (\( x \sim y \)) and (\( y \sim z \)) implies not (\( y \sim z \)), but there are cases where analogy is transitive: (\( x \sim y \)) and (\( y \sim z \)) \( \Rightarrow \) \( x \sim z \). If the analogy is transitive, it is called contagious and is an equivalence relation.

The analogy is based on process modelling. An object \( x \) belonging to set \( C \) subset of or above another object is modelling another object \( y \in O \), if \( x \) is contagious analogy \( y \). Symbol which indicates that the shape of \( y \) is \( x \sim y \). The relationship \( \sim \) is a binary relationship with \( A \cup C \) in scope, and as co field \( O \). Relationship modelling has the following properties:

- is nonsimmetrical (original model can model or not);
• is reflexive (every object is its own model);
• is transitive (imitation is contagious).

As a transitive and reflexive, is a relationship pre-order modelling, so it is stronger than the analogy. The model is based on modelling the system that is reality.

In economic practice, an econometric model is used to explain the phenomenon resultive change in relation to changes in y or x factor, to estimate the expected values of the phenomenon y (simulating it) of possible values that can record economic factor x, and Finally, the forecast according to the values y phenomenon x, the forecast interval \( v, v = 1,2, ..., h \).

To allow users to verify (performance) obtained an econometric model, it must be presented with the following information:

\[
\hat{y}_i = \hat{\alpha} + \hat{\beta}x_i
\]

- R-Correlation report
- d-Durbin-Watson variable

\( S_\hat{\alpha} \) - Deviation of the residual variable

With this information we can test:

- Error independence \( \rightarrow \) "d" test - Durbin – Watson;
- Estimators significance \( \rightarrow \) "F" test;
- Model similarity \( \rightarrow \) "F" test.

Multiple equation models are used primarily to substantiate a firm business plan in this case, control variables - that those phenomena can be established, controlled or regulated by managerial decisions - salaries, taxes, customs fees, interest on types of loans etc. - determine those possible values, resulting from an economic analysis, or statistically probable, which introduced the multiple equation model, will generate confidence intervals of the endogenous variables, the scenarios that will result from one or some economic decisions.

Also, not infrequently, endogenous variables are not expected from solving the econometric model, but through surveys of managers on the values we expect for the economic phenomena that may influence a greater or lesser extent.

3. Case Study - Using the model with multiple equations. Keynes’s dynamic model

Considering the many possibilities of use of multiple equations models with firm management to substantiate the financial policies we chose to develop a small case study whose objective is to characterize the economic development of the CNFR NAVROM SA with an econometric model of equations multiple. It is based dynamic key financial indicators in the period 1991-2010, presented in the table below:

<table>
<thead>
<tr>
<th>Years</th>
<th>Total Expenses ((\text{Lei}))</th>
<th>Net Profit ((\text{Lei}))</th>
<th>Profit tax ((\text{Lei}))</th>
<th>Total Income ((\text{Lei}))</th>
<th>(v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991</td>
<td>67,39</td>
<td>3,22</td>
<td>0,61</td>
<td>71,22</td>
<td>-</td>
</tr>
<tr>
<td>1992</td>
<td>70,94</td>
<td>3,39</td>
<td>0,65</td>
<td>74,97</td>
<td>71,22</td>
</tr>
<tr>
<td>1993</td>
<td>35,94</td>
<td>-1,96</td>
<td>0,00</td>
<td>33,99</td>
<td>74,97</td>
</tr>
<tr>
<td>1994</td>
<td>27,64</td>
<td>1,38</td>
<td>0,00</td>
<td>29,02</td>
<td>33,99</td>
</tr>
<tr>
<td>1995</td>
<td>25,31</td>
<td>1,09</td>
<td>0,17</td>
<td>26,58</td>
<td>29,02</td>
</tr>
<tr>
<td>1996</td>
<td>20,79</td>
<td>2,81</td>
<td>0,53</td>
<td>24,13</td>
<td>26,58</td>
</tr>
<tr>
<td>1997</td>
<td>17,94</td>
<td>2,42</td>
<td>0,46</td>
<td>20,82</td>
<td>24,13</td>
</tr>
<tr>
<td>1998</td>
<td>18,10</td>
<td>1,51</td>
<td>0,29</td>
<td>19,90</td>
<td>20,82</td>
</tr>
<tr>
<td>1999</td>
<td>20,55</td>
<td>-1,52</td>
<td>0,00</td>
<td>19,03</td>
<td>19,90</td>
</tr>
<tr>
<td>2000</td>
<td>32,35</td>
<td>1,08</td>
<td>0,00</td>
<td>33,43</td>
<td>19,03</td>
</tr>
<tr>
<td>2001</td>
<td>44,73</td>
<td>0,08</td>
<td>0,11</td>
<td>44,92</td>
<td>33,43</td>
</tr>
<tr>
<td>2002</td>
<td>41,80</td>
<td>0,18</td>
<td>0,29</td>
<td>42,27</td>
<td>44,92</td>
</tr>
<tr>
<td>2003</td>
<td>44,02</td>
<td>0,56</td>
<td>0,24</td>
<td>44,82</td>
<td>42,27</td>
</tr>
<tr>
<td>2004</td>
<td>52,56</td>
<td>0,05</td>
<td>0,04</td>
<td>52,66</td>
<td>44,82</td>
</tr>
<tr>
<td>2005</td>
<td>59,94</td>
<td>0,46</td>
<td>0,13</td>
<td>60,53</td>
<td>52,66</td>
</tr>
<tr>
<td>2006</td>
<td>59,39</td>
<td>0,59</td>
<td>0,07</td>
<td>60,04</td>
<td>60,53</td>
</tr>
<tr>
<td>2007</td>
<td>61,47</td>
<td>0,98</td>
<td>0,35</td>
<td>62,80</td>
<td>60,04</td>
</tr>
<tr>
<td>2008</td>
<td>73,90</td>
<td>3,55</td>
<td>0,50</td>
<td>77,95</td>
<td>62,80</td>
</tr>
<tr>
<td>2009</td>
<td>56,43</td>
<td>2,21</td>
<td>0,44</td>
<td>59,09</td>
<td>77,95</td>
</tr>
<tr>
<td>2010</td>
<td>44,03</td>
<td>1,26</td>
<td>0,24</td>
<td>45,53</td>
<td>59,09</td>
</tr>
<tr>
<td>Total</td>
<td>875,21</td>
<td>23,37</td>
<td>5,12</td>
<td>905,70</td>
<td>885,17</td>
</tr>
</tbody>
</table>
Based on data from the table above, the characterization of economic development during the 1991-2010 of CNFR NAVROM SA can be done using a multi-equation econometric model whose structural form is:

\[
\begin{align*}
C_t &= a_0 + a_1 V_t + u_t \\
PN_t &= b_0 + b_1 V_t + b_2 V_{t-1} + u_2 \\
V_t &= C_t + PN_t + IMP_t
\end{align*}
\] (1) (2) (3)

The above presents several features such as:
- The first equation is a behavioural relationship of total expenditure;
- Equation (2) is also a behavioural relationship, describing the net profit / net loss, but with a dynamic offset due to the inclusion of variables that gives this feature and the model with multiple equations;
- Equation (3) represents a relationship of economic identity.

The model contains five variables with multiple equations, three of which are endogenous variables \( (C_t, PN_t, V_t) \) and two exogenous variables \( (IMP_t, V_{t-1}) \).

Econometric Analysis of the model with multiple equations reveals that the model is over identified because:
- In equation (1), the number of missing variables is equal to 3, higher number endogenous variables minus one (3-1), which means that the equation is over identified;
- Even if equation (2) is correctly identified \( (2 = 3-1) \), the model is over identified because equation (1).

With this feature, model parameter estimation is done using MCMMP implemented in two phases [11].

**Phase I**, \( V_t \) variable, which is endogenous in equation (3), but exogenous in equations (1) and \( IMP_t \) (2) will fall back according to the exogenous variables, and the resulting equation:

\[ V_t = \alpha + \beta \cdot V_{t-1} + \phi \cdot IMP_t + w_t \]

Is applying M.C.M.M.P. to estimate the parameters \( \alpha, \beta, \phi \) and calculating estimated values of the variable \( \hat{V}_t \):

\[
F(\hat{\alpha}, \hat{\beta}, \hat{\phi}) = \min \sum_{t=2}^{n} \left( V_t - \hat{\alpha} - \hat{\beta} V_{t-1} - \hat{\phi} IMP_t \right)^2
\]

\[
F(\hat{\alpha}) = 0 \Rightarrow \sum_{t=2}^{n} \left( V_t - \hat{\alpha} - \hat{\beta} V_{t-1} - \hat{\phi} IMP_t \right) (1) = 0
\]

\[
(n-1)\hat{\alpha} + \beta \sum_{t=2}^{n} V_{t-1} + \phi \sum_{t=2}^{n} IMP_t = \sum_{t=2}^{n} V_t
\]

\[
F(\hat{\beta}) = 0 \Rightarrow \sum_{t=2}^{n} \left( V_t - \hat{\alpha} - \hat{\beta} V_{t-1} - \hat{\phi} IMP_t \right) (2) = 0
\]

\[
\hat{\alpha} \sum_{t=2}^{n} V_{t-1} + \hat{\beta} \sum_{t=2}^{n} V_{t-1}^2 + \hat{\phi} \sum_{t=2}^{n} IMP_t V_{t-1} = \sum_{t=2}^{n} V_t V_{t-1}
\]

\[
F(\hat{\phi}) = 0 \Rightarrow \sum_{t=2}^{n} \left( V_t - \hat{\alpha} - \hat{\beta} V_{t-1} - \hat{\phi} IMP_t \right) (3) = 0
\]

\[
\hat{\alpha} \sum_{t=2}^{n} IMP_t + \hat{\beta} \sum_{t=2}^{n} IMP_t V_{t-1} + \hat{\phi} \sum_{t=2}^{n} IMP_t^2 = \sum_{t=2}^{n} V_t IMP_t
\]

hence the system of normal equations:

\[
\begin{align*}
(n-1)\hat{\alpha} + \hat{\beta} \sum_{t=2}^{n} V_{t-1} + \hat{\phi} \sum_{t=2}^{n} IMP_t &= \sum_{t=2}^{n} V_t \\
\hat{\alpha} \sum_{t=2}^{n} V_{t-1} + \hat{\beta} \sum_{t=2}^{n} V_{t-1}^2 + \hat{\phi} \sum_{t=2}^{n} IMP_t V_{t-1} &= \sum_{t=2}^{n} V_t V_{t-1} \\
\hat{\alpha} \sum_{t=2}^{n} IMP_t + \hat{\beta} \sum_{t=2}^{n} IMP_t V_{t-1} + \hat{\phi} \sum_{t=2}^{n} IMP_t^2 &= \sum_{t=2}^{n} V_t IMP_t
\end{align*}
\]
\[
\begin{cases}
19\hat{a} + 858,1716,3\hat{b} + 4,5086 \hat{\phi} = 832,4831 \\
\Rightarrow 858,1716\hat{a} + 45654,8430\hat{b} + 222,7958\hat{\phi} = 42,657,1034 \\
4,5086\hat{a} + 222,7958\hat{b} + 1,8289 \hat{\phi} = 221,0521
\end{cases}
\]

To estimate the parameters \( \hat{a}, \hat{b}, \hat{\phi} \) and for \( \hat{V}_i \) values we used Excel’s and results are the following:

\[
\begin{align*}
\hat{a} &= 43,5473 \\
\hat{b} &= -0,0645 \\
\hat{\phi} &= 13,3993 \\
\hat{V}_1 &= 43,5473 - 0,0645 \cdot V_{t-1} + 13,3993 \cdot \text{IMP}_t
\end{align*}
\]

**Phase II.** Since \( V_t \), endogenous variable (see equation (3)) is an exogenous variable in equations (1) and (2) in these equations it will introduce its estimate in Phase I \( \hat{V}_t \).

Parameter estimation equations (1) and (2) will be made with MCMMP applied to each equation:

\[
\begin{align*}
C_i &= a_0 + a_1 \cdot \hat{V}_i + u_i \\
\text{PN}_i &= b_0 + b_1 \cdot \hat{V}_i + b_2 \cdot V_{t-1} + u_2
\end{align*}
\]

Equation estimators (1), \( \hat{a}_0 \) and \( \hat{a}_1 \), M.C.M.M.P. resulting from the application:

\[
\begin{align*}
F\left(\hat{a}_0, \hat{a}_1\right) &= \min \left[ \sum_{i=2}^{20} \left( C_i - \hat{a}_0 - \hat{a}_1 \hat{V}_i \right)^2 \right] \\
F\left(\hat{a}_0\right) &= 0 \Rightarrow (n-1)\hat{a}_0 + \hat{a}_1 \sum_{i=2}^{20} \hat{V}_i = \sum_{i=2}^{20} C_i \\
F\left(\hat{a}_1\right) &= 0 \Rightarrow \hat{a}_1 \sum_{i=2}^{20} \hat{V}_i + \hat{a}_1 \sum_{i=2}^{20} \hat{V}_i^2 = \sum_{i=2}^{20} C_i \hat{V}_i
\end{align*}
\]

Calculations were performed using Excel to derive the following values:

\[
\begin{align*}
C_i &= \hat{a}_0 + \hat{a}_1 \cdot \hat{V}_i = 0.2024 + 0.9640 \cdot \hat{V}_i; \\
R_i &= 0.9966 \quad d=1.4213 \\
s_{\hat{a}_0} &= 1.5662 \\
s_{\hat{a}_1} &= (0.9215) \\
s_{\hat{b}_0} &= (0.0189)
\end{align*}
\]

Preceding similarly with equation (2) all calculations was performed using Excel to derive the following values:

\[
\begin{align*}
\hat{V}_i &= \hat{b}_0 + \hat{b}_1 \cdot \hat{V}_i + \hat{b}_2 \cdot V_{t-1} = -28.478,8001 + 688,5080 V_i + 23,4956 V_{t-1}; \\
R_i &= 1,5287 \quad s_{\hat{b}_0} = 14.092,7620 \quad d=0, 4861 \\
s_{\hat{b}_1} &= (40,2805) \\
s_{\hat{b}_2} &= (2.2759) \\
s_{\hat{b}_3} &= (0.6909)
\end{align*}
\]

Checking the significance of the estimators and the model will be made for each equation separately.
• Equation (1)

\( \hat{a}_0, \hat{a}_1 \) estimators are significantly different from zero, with a significance level \( \alpha = 0.05 \), if it verifies the following relations:

\[
\frac{\hat{a}_0}{s_{\hat{a}_0}} > t_{n,n-k-1} \leftrightarrow \frac{0.2024}{0.9215} = 0.2197 < t_{0.05,17} = 2.11
\]

\[
\frac{\hat{a}_1}{s_{\hat{a}_1}} = \frac{0.9640}{0.0189} = 51.1041 > t_{0.05,17} = 2.11
\]

Following the above calculations it appears that only is significantly different from zero, with a significance level \( \alpha = 0.05 \), \( \hat{A}_0 \), is insignificant.

To verify the hypothesis of independence of error test applies Durbin - Watson, which involves calculating the variable \( d \) using the relationship:

\[
d = \frac{\sum_{i=1}^{n} (\hat{u}_i - \hat{u}_{i-1})^2}{\sum_{i=2}^{n} \hat{u}_{i-1}^2} = 1.4213
\]

The distribution table Durbin - Watson, according to a significance level of \( \alpha = 0.05 \), the number of explanatory variables \( k = 1 \) and \( n \) = number of observations taken over 19 theoretical values \( d_1 \) and \( d_2 = 1.40 = 1.18 \). Comparing the calculated value of the variable \( d \) with the two theoretical values is found that \( 0 < d_2 = 1.40 < d = 1.4213 < 2 - d_2 = 2.60 \), range for which no error autocorrelation.

Checking the significance of the correlation ratio:

\[
F_r = \frac{(n - k - 1) R^2}{1 - R^2} > F_{0.05, n-k-1}
\]

\[
F_{0.05,1,17} = 4.45
\]

\[
F_r = 17 \frac{0.9931}{1 - 0.9931} = 2.466,5393 > F_{0.05,1,17} = 4.45
\]

Report of the correlation is significantly different from zero, with a significance level \( \alpha = 0.05 \). After applying t tests, \( F \) and \( d \) shows that the model played by equation (1) is significant (valid).

• Equation (2)

Checking the significance of the estimators:

\[
\frac{\hat{b}_0}{s_{\hat{b}_0}} > t_{n,n-k-1} \leftrightarrow \frac{-28.478,8001}{40.2804} = 707.0124 > t_{0.05,16} = 2.12
\]

\[
\frac{\hat{b}_1}{s_{\hat{b}_1}} > t_{n,n-k-1} \leftrightarrow \frac{688.5080}{2.2759} = 302.5177 > t_{0.05,16} = 2.12
\]

\[
\frac{\hat{b}_2}{s_{\hat{b}_2}} > t_{n,n-k-1} \leftrightarrow \frac{23.4956}{0.6909} = 34.0043 > t_{0.05,16} = 2.12
\]

\( \hat{b}_0, \hat{b}_1, \) and \( \hat{b}_2 \) estimators are significantly different from zero, with a significance level \( \alpha = 0.05 \).

To verify the hypothesis of independence of error test applies Durbin - Watson, which involves calculating the variable \( d \) using the relationship:
\[ d' = \sum_{t=2}^{n} \left( \hat{u}_{2t} - \hat{u}_{2t-1} \right)^2 / \sum_{t=2}^{n} \hat{u}_{2t}^2 = 0.4861 \]

The distribution table Durbin - Watson, according to a significance level of \( \alpha = 0.05 \), the number of explanatory variables \( k = 2 \) and \( n \) = number of observations taken over 19 theoretical values \( d_1 \) and \( d_2 = 1.53 = 1.08 \). Comparing the calculated value of the variable \( d \) with the two theoretical values is found that \( 0 < d' = 0.4861 < d_1 = 1.08 \), which correspond to the range of positive autocorrelation.

Checking the significance of the correlation ratio:

\[ F_x = \frac{n - k - 1}{k} \frac{R_1^2}{1 - R_x^2} \geq F_{u,k,(n-k-1)} \]

\[ F_{0.05;2.18} = 3.63 \]

\[ F_x = 13.9843 > F_{0.05;2.18} = 3.63 \]

Report of the correlation is significantly different from zero, with a significance level \( \alpha = 0.05 \).

Conclusions

Description of the financial situation of CNS NAVROM SA during 1991-2010 can be done directly with the multiplier effect.

Multipliers with direct or structural parameters of the model are represented by the structural form:

- \( a_1 \) - the marginal rate of total expenditure, showing that period, an increase of one hundred lei profit tax expenses increased by approximately one hundred 0.96 lei.
- \( b_1 \) - marginal net profit, which expresses the fact that period, it increased by one hundred lei 746.08 to an increase of one hundred lei profit tax.
- \( b_2 \) - reveals that the tax paid in the previous year had a positive effect on growth in net profit that rose by about 23.50 lei to one hundred percent increase with a lei of tax from the previous year.

References