Stochastic Simulation of the Exchange Rate

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Abstract. The rational expectations paradigm, that dominates macroeconomics fails to take into account the complexity of the information, which is so vast that the individual brain cannot understand the full of it. The agents are boundedly rational, so they use simple forecasting rules that do not incorporate all available information, but they are willing to learn and will switch to other rules if it turns out that these rules are more profitable than the rule they have been using. Such trial and error learning strategies create the dynamics in the foreign exchange market, with two types of equilibria, a fundamental and a non-fundamental equilibrium to which the exchange rate is attracted.

Keywords: behavioral finance, rational expectations, fundamental exchange rate, non-fundamental equilibrium

1. Introduction

Since the start of the rational-expectations revolution, macroeconomic analysis has been dominated by the assumption of the rational representative agent. The representative agent is assumed to continuously maximize his utility in an intertemporal framework. The forecasts made by this agent are rational in the sense that they take all available information into account, including the information embedded in the structure of the model. This implies that agents do not make systematic errors in forecasting future variables. The great attractiveness of the rational-expectations model is that it imposes consistency between the agent’s forecasts (the subjective probability distribution of future variables) and the forecasts generated by the model (the objective probability). The fact that markets are efficient means that asset prices reflect all relevant information about the fundamental variables that determine the value of the asset. The mechanism that ensures efficiency can be described as follows: when rational agents value a particular asset, they compute the fundamental value of that asset and price it accordingly. If they obtain new information, they will immediately incorporate that information in their valuation of the asset.

But, the rational expectations efficient market model (REEM) of the foreign exchange market it is not consistent with the empirical testing. An alternative model was presented by De Grauwe and Grimaldi (2006). They started from the observation that the information is so complex that no single agent is capable of fully understanding it. Agents are aware of the exceptional complexity of the world in which they live. They will therefore follow a different forecasting strategy than the one the rational expectations model assumes. Instead, the strategy they follow consists of two steps. First, agents apply simple forecasting rules. This is often referred to as “heuristic rules” in the psychological literature. These rules, by necessity, only use small parts of the full (but too complex) information set available in the world. Although
agents have only a limited capacity for understanding the world, they want to find out whether the rule they use, is a good one. The way they do this is by checking “ex post” how profitable the rule is compared to other available rules. If they find out that the rule is less profitable, they will consider switching to the better rule. If not, they stick to their initial rule. Such trial and error strategies are favored by individual agents when they face an environment that they do not understand well and try to learn its complexity. In this sense trial and error is a learning strategy.

In the real world agents are “boundedly rational”, that means that because individual agents have a limited ability to process and to analyze the available information, they are forced to select simple forecasting rules. This is the “bounded” part in their rationality. These agents, however, exhibit rational behavior in the sense that they check the profitability of these rules ex post and are willing to switch to the more profitable one. This assumption of bounded rationality was first proposed by Simon(1955), and later, it was developed by researchers of the “behavioral economics” school which uses concepts from psychology.

2. De Grauwe and Grimaldi’s behavioural finance model of the exchange rate

Using these insights, De Grauwe and Grimaldi developed a simple exchange rate model. They started by defining the concept of fundamental exchange rate as the exchange rate that is consistent with equilibrium in the real economy (in a very simple model this could be the Purchasing Power Parity-value of the exchange rate). They assumed that the fundamental exchange rate, \( s_t^* \), is exogenous and that it behaves like a random walk without drift:

\[
S_t^* = S_{t-1}^* + \varepsilon_t
\]

where \( \varepsilon_t \) is a white noise error term.

They assumed that agents can use two types of simple forecasting rules. One type of forecasting rule will be called fundamentalist, and agents who use this rule will be called fundamentalists. The second type of rule will be called chartist and the agents who use this rule will be named chartists. In theory it is used, also the term ‘technical analysts’. The fundamentalists are assumed to know the fundamental exchange rate. They compare the present market exchange rate with the fundamental rate and they forecast the future market rate to move towards the fundamental rate. In this sense they follow a negative feedback rule:

\[
E_{f,t}(\Delta s_{t+1}) = -\psi(s_t - s_t^*)
\]

where \( E_{f,t} \) is the forecast made in period \( t \) by the fundamentalists using information up to time \( t \), \( s_t \) is the exchange rate in period \( t \), \( \Delta s_{t+1} \) is the change in the exchange rate, and \( \psi > 0 \) measures the speed with which the fundamentalists expect the exchange rate to converge to the fundamental one. This parameter may be related to the speed of adjustment of prices in the goods market. The chartists are assumed to follow a positive feedback rule, so they extrapolate past movements of the exchange rate into the future. The authors assume that chartists extrapolate only last period’s exchange rate into the future. The chartists’ forecast is:

\[
E_{c,t}(\Delta s_{t+1}) = \beta \Delta s_t
\]

where \( E_{c,t} \) is the forecast made by the chartists using information up to time \( t \), and \( \beta (0 < \beta < 1) \) is the coefficient expressing the degree with which chartists extrapolate the past change in the exchange rate. The chartists do not take into account information concerning the
fundamental exchange rate. In this sense they can be considered to be noise traders.

Agents use one of the two rules, compare their profitability ex post and then decide whether to keep the rule or switch to the other one. This idea was implemented using a fitness criterion in the spirit of Brock and Hommes (1997), (1998) which is based on discrete choice theory. This means that the fractions of the total population of agents using chartist and fundamentalist rules are a function of the relative (risk adjusted) profitability of these rules:

\[
w_{f,t} = \frac{\exp \gamma \pi_{f,t}}{\exp \gamma \pi_{f,t} + \exp \gamma \pi_{c,t}} \tag{4}
\]
\[
w_{c,t} = \frac{\exp \gamma \pi_{c,t}}{\exp \gamma \pi_{f,t} + \exp \gamma \pi_{c,t}} \tag{5}
\]

where \(w_{f,t}\) and \(w_{c,t}\) are the fractions of the population who use fundamentalist, respectively chartist forecasting rules. Obviously \(w_{f,t} + w_{c,t} = 1\). The variables \(\pi_{f,t}\) and \(\pi_{c,t}\) are the (risk adjusted) profits realized by the use of chartists’ and fundamentalists’ forecasting rule in period \(t\), \(\pi_{f,t} = \pi_{f,t} - \mu \sigma_{f,t}^2\) and \(\pi_{c,t} = \pi_{c,t} - \mu \sigma_{c,t}^2\), and \(\pi_{f,t}\) and \(\pi_{c,t}\) are the profits made in forecasting, while \(\sigma_{f,t}^2\) and \(\sigma_{c,t}^2\) are variables expressing the risks chartists and fundamentalists incur when making forecasts. As a measure of this risk they consider the forecast errors and \(\mu\) is the coefficient of risk aversion.

Equations 4 and 5 suggest that the parameter \(\gamma\) measures the intensity with which the technical traders and fundamentalists revise their forecasting rules. With an increasing \(\gamma\) agents react strongly to the relative profitability of the rules. In the limit when \(\gamma\) goes to infinity all agents choose the forecasting rule which proves to be more profitable. When \(\gamma\) is equal to zero agents are insensitive to the relative profitability of the rules and the fraction of technical traders and fundamentalists is constant and equal to 0.5.

The authors define the profits as the one-period earnings of investing $1 in the foreign asset:

\[\pi_{i,t} = \left[ s_t - s_{t-1} \right] \text{sgn}(E_{i,t} - s_{t-1}) \]  \tag{6}

where \(\text{sgn}[x] = \begin{cases} 1, & \text{ptr.x} > 0 \\ 0, & \text{ptr.x} = 0 \\ -1, & \text{ptr.x} < 0 \end{cases},\) and \(i = c, f\)

Thus, when agents forecasted an increase in the exchange rate and this increase is realized, their per unit profit is equal to the observed increase in the exchange rate. If instead the exchange rate declines, they make a per unit loss which equals this decline, because in this case they have bought foreign assets which have declined in price. The risk associated with forecasting is the forecast error, and agents look just at last period’s forecast error:

\[\sigma_{i,t}^2 = \left[ E_{i,t+1} - s_{t+1} \right]^2 \]  \tag{7}

The forecast at market level is obtained by aggregating the chartist and fundamentalist forecasts:

\[E_t \Delta s_{t+1} = -w_{f,t} \psi(s_t - s_t^*) + w_{c,t} \beta \Delta s_t \]  \tag{8}

The realized change in the market exchange rate in period \(t+1\) is equal with the market forecast made at time \(t\) plus a white noise error, that occur in period \(t+1\) (which includes the information that could not be predicted at time \(t\)):
\[ \Delta s_{t+1} = -w_f \psi (s_t - s_t^*) + w_c \beta \Delta s_t + \xi_{t+1} \quad (9) \]

3. Stochastic simulation of the model in Eviews

We defined the model in Eviews by choosing **Objects/ New Object…Model**, and added one by one the equations, which can be identities or stochastic equations.

In a stochastic simulation, the equations of the model are solved so that they have residuals which match to randomly drawn errors, and, optionally, the coefficients and exogenous variables of the model are also varied randomly. In this case, the model generates a distribution of outcomes for the endogenous variables in every period. A stochastic simulation follows a sequence, with the following differences:

- When binding the variables, a temporary series is created for every endogenous variable in the model. Additional series in the work file are used to hold the statistics for the tracked endogenous variables.
The model is solved repeatedly for different draws of the stochastic components of the model. If coefficient uncertainty is included in the model, then a new set of coefficients is drawn before each repetition. During the repetition, errors are generated for each observation in accordance with the residual uncertainty and the exogenous variable uncertainty in the model. At the end of each repetition, the statistics for the tracked endogenous variables are updated to reflect the additional results.

The Solution algorithm box lets us select the algorithm that will be used to solve the model for a single period. The following choices are available:

- **Gauss-Seidel**: the Gauss-Seidel algorithm is an iterative algorithm, where at each iteration it solves each equation in the model for the value of its associated endogenous variable, treating all other endogenous variables as fixed.
- **Newton**: Newton's method is also an iterative method, where at each iteration is taken a linear approximation to the model, and then the linear system is solved to find a root of the model. This algorithm can handle a wider class of problems than Gauss-Seidel, but requires considerably more working memory and has a much greater computational cost when applied to large models.

After the simulation is run, the results can be seen in suggestive graphs.
Varying the parameters of the model, we can conclude regarding to the evolution of exchange market rate towards the fundamental rate.

Stochastic shocks occur in the model because the fundamental exchange rate is driven by a random walk. In two different simulation runs, using the same parameter configurations ($\psi=0.2$, $\beta=0.9$, $\gamma=5$), we obtained that the exchange rate is very often disconnected from the fundamental exchange rate:

![Graphs showing the relationship between fundamental and market rates](image)

The market exchange rate follows movements that are dissociated from the fundamental rate. Grimaldi and De Grauwe separated these exchange rate movements in two regimes. In one regime the exchange rate follows the fundamental exchange rate quite closely, so they called “fundamental regimes”, which alternate with regimes called “bubble” or non-fundamental regimes, corresponding to the situations in which the chartists’ weights are very close to 1. In contrast, fundamental regimes are those during which the chartist’s weights are below 1 and fluctuating significantly.

The fundamental and non-fundamental regimes alternate in unpredictable ways. Also, if we plot the one period changes of the simulated exchange rate and of the fundamental rate, as the authors of the model suggested, we find that the exchange rate is subject to much more short-term volatility than the fundamental exchange rate. In addition, it appears that the exchange rate is occasionally subject to very large changes. Fundamental rate follows a random walk, so the one period changes are normally distributed. The changes of the simulated market exchange rate are larger on average, but more importantly there are regularly very large outliers, which suggest that the changes of the market exchange rate are not normally distributed.
This sensitivity can be illustrated with simulations based on different parameter values. The results of stochastic simulations of the model for different values of $\gamma$ are presented in Fig. 3. Parameter $\gamma$ measures the sensitivity of the switching rule to risk adjusted profits. When $\gamma$ is high agents react strongly to changing profitabilities of the forecasting rules they have been using. Conversely when $\gamma$ is small they do not let their forecasting rules depend much on these relative profitabilities. From fig.3 results that, when $\gamma$ is large, the exchange rate tends to deviate strongly from the fundamental value, being attracted most of the time by non-fundamental equilibrium. When $\gamma$ is low, agents are not very sensitive to relative profitabilities and the exchange rate is attracted by the fundamental equilibrium most of the time.
Another important parameter of the model is $\beta$, the degree of extrapolation, used by chartist in making the forecasts. In fig.4 can be seen that when $\beta$ is high, the exchange rate is strongly attracted by non-fundamental equilibrium. When $\beta$ is small the forces of attraction of the fundamental variable are very strong, so that the exchange rate remains very close to its fundamental value.
So, we can say that, as $\beta$ increases, the probability of obtaining a bubble equilibrium increases.

**Conclusions**

Empirical studies suggest that extrapolative forecasting rules, which do not take into account information about the fundamental exchange rate, can predict well the dynamics of the market rate, and on average create more profits than fundamental-based forecasting rules. The reason why this happens is that these extrapolative rules create noise that generates additional profits in a self-fulfilling way. As a result, these are forecasting strategies that can survive in the long run.

**References**


